UNIT-V

8.1 SHEAR AND PRINCIPAL STRESSES

The shear distribution in an uncracked structural concrete member for which the deformation is assumed to be linear is a function of the shear force and the properties of the cross-section of the member. The shear stress at a point is expressed as,

$$\tau_v = \left(\frac{VS}{Ib}\right)$$

where τ_v = shearing stress due to transverse loads

V = shearing force

S = statical moment (first moment of area)

I = second moment of area of section about its centriod

b = breadth of section at the given point

The maximum and minimum principal stresses developed are given by,

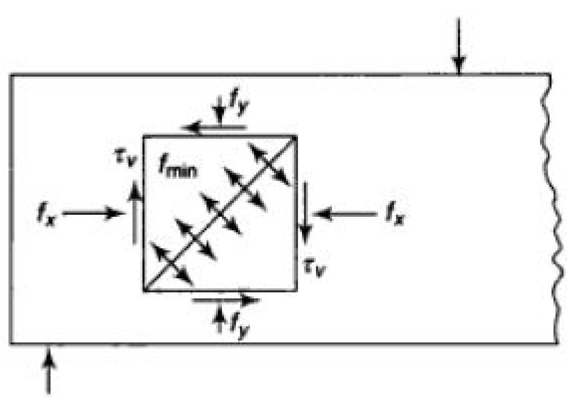


Fig. 8.1 *Principal Tensile Stresses in a Prestressed Member*

In general, there are three ways of improving the shear resistance of structual concrete members by prestressing techniques:

1. Horizontal or axial prestressing;
2. Prestressing by inclined or sloping cables; and
3. Vertical or transverse prestressing.

The effect of these different techniques of prestressing on the magnitude of principal tensile stress is illustrated by the following examples.

EXAMPLE 8.1 A prestressed concrete beam (span = 10 m) of rectangular section, 120 mm wide and 300 mm deep, is axially prestressed by a cable carrying an effective force of 180 kN. The beam supports a total uniformly distributed load of 5 kN/m which includes the self-weight of the member. Compare the magnitude of the principal tension developed in the beam with and without the axial prestress.

$$A = (120 \times 300) = 36 \times 10^3 \text{ mm}^2$$

$$I = 27 \times 10^7 \text{ mm}^4$$

$$w_d = 5 \text{ kN/m}$$

$$\text{Shear force at support, } V = \left(\frac{5 \times 10}{2}\right) = 25 \text{ kN}$$

$$\text{Maximum shear stress at support, } \tau_v = \left(\frac{3V}{2bh}\right) = \left(\frac{3}{2} \times \frac{25 \times 10^3}{120 \times 300}\right)$$

$$= 1.05 \text{ N/mm}^2$$

$$\text{Principal stresses} = \pm\frac{1}{2}\sqrt{4\tau_v^2} = \pm\tau_v = 1.05 \text{ N/mm}^2$$

(compression and tension)

$$\text{Axial prestress, } f_x = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = 5 \text{ N/mm}^2$$

$$\text{Maximum and minimum principal stress} = \left(\frac{f_x}{2}\right) \pm \frac{1}{2}\sqrt{f_x^2 + 4\tau_v^2}$$

$$= \left(\frac{5}{2}\right) \pm \frac{1}{2}\sqrt{5^2 + 4 \times 1.05^2}$$

$$= 2.5 \pm 2.73$$

$$= +5.23 \text{ N/mm}^2 \text{ (compression)}$$

$$= -0.23 \text{ N/mm}^2 \text{ (tension)}$$

Hence with axial prestress, the principal tension is reduced by,

$$\left(\frac{1.05 - 0.23}{1.05}\right) \times 100 = 78\%$$

EXAMPLE 8.2 For the beam in Example 8.1, instead of axial prestresing, a curved cable having an eccentricity of 100 mm at the centre of span and reducing to zero at the supports is used, the effective force in the cable being 180 kN. Estimate the percentage reduction in the principal tension in comparison with the case of axial prestressing.

$$\text{Slope of cable at support} = \left(\frac{4e}{L}\right) = \left(\frac{4 \times 100}{10 \times 1000}\right) = 0.04 \text{ radians}$$

Vertical component of the prestressing force = $(180 \times 0.04) = 7.2$ kN
Horizontal component of the prestresssing force = 180 kN

$\therefore$ Net shear at support, $V = (25 - 7.2) = 17.80$ kN

$$\text{Maximum shear stress} = \left(\frac{3}{2}\frac{V}{bh}\right) = \frac{3}{2}\left(\frac{17.8 \times 10^3}{120 \times 300}\right)$$
$$= 0.74 \text{ N/mm}^2$$

Axial prestress,
$$f_x = \left(\frac{180 \times 10^3}{120 \times 300}\right) = 5 \text{ N/mm}^2$$

$\therefore$
$$f_{\substack{max\\min}} = \frac{5}{2} \pm \frac{1}{2}\sqrt{5^2 + 4 \times 0.74^2} = (2.5 \pm 2.62)$$
$$= +5.12 \text{ N/mm}^2 \text{ (compression)}$$
$$= -0.12 \text{ N/mm}^2 \text{ (tension)}$$

In comparison with axial prestressing, the percentage reduction in principal tensile stress is,

$$\left(\frac{0.23 - 0.12}{0.23}\right) \times 100 = 48\%$$

When compared with a beam without any prestress, the percentage reduction in principal tension is

$$\left(\frac{1.05 - 0.12}{1.05}\right) \times 100 = 88.5\%$$

EXAMPLE 8.5 A post-tensioned beam of rectangular cross-section, 200 mm wide and 400 mm deep, is 10 m long and carries an applied load of 8 kN/m, uniformly distributed on the beam. The effective prestressing force in the cable is 500 kN. The cable is parabolic with zero eccentricity at the supports and a maximum eccentricity of 140 mm at the centre of span.

(a) Calculate the principal stresses at the supports.

(b) What will be the magnitude of the principal stresses at the supports in the absence of prestress?

$$
\begin{aligned}
g &= 1.92 \text{ kN/m},\\
q &= 8 \text{ kN/m},\\
P &= 500 \text{ kN},\\
e &= 140 \text{ mm, and}\\
L &= 10 \text{ m}
\end{aligned}
$$

Slope of cable at support, $\theta = \left(\frac{4e}{L}\right) = \left(\frac{4 \times 140}{10 \times 1000}\right) = 0.056$ radians

Vertical component $= P \sin \theta = (500 \times 0.056) = 28$ kN

Horizontal component $= P \cos \theta = (500 \times 0.998) = 500$ kN

Maximum shear at support due to loads $= (8.0 + 1.92)10/2 = 49.60$ kN

Net shear at support $= 49.6 - 28.0 = 21.6$ kN

Shear stress, $\tau_v = \left(\frac{3V}{2bh}\right) = \frac{3}{2}\left(\frac{21.6 \times 10^3}{200 \times 400}\right) = 0.405 \text{ N/mm}^2$

Direct stress $= \left(\frac{500 \times 10^3}{200 \times 400}\right) = 6.25 \text{ N/mm}^2$

(a) Maximum and minimum principal stresses

$$= \left(\frac{6.25}{2} \pm \frac{1}{2}\sqrt{6.25^2 + 4 \times 0.405^2}\right) = 6.275 \text{ N/mm}^2 \text{ (compression)}$$

$$= -0.025 \text{ N/mm}^2 \text{ (tension)}$$

(b) In the absence of prestress, vertical shear $= 49.60$ kN

Maximum and minimum principal stresses = Maximum shear stress

$$= \left(\frac{3}{2} \times \frac{49.6 \times 10^3}{200 \times 400}\right) = 0.93 \text{ N/mm}^2$$

8.2.1 Types of Shear Cracks

Research over the years have shown that there are two major modes of shear cracking in structural concrete beams[3, 4]. These two types, generally referred to as web-shear and flexure-shear cracks, are illustrated in Fig. 8.2. Web-shear cracks generally start from an interior point, when the local principal tensile stress exceeds the tensile strength of concrete. Web-shear cracks are likely to develop in highly prestressed beams with thin webs[5], particularly when the beam is subjected to large concentrated loads near a simple support. Flexure-shear cracks are first initiated by flexural cracks in the inclined direction. Flexure-shear cracks develop when the combined shear and flexural tensile stresses produce a principal tensile stress exceeding the tensile strength of concrete. In members without shear reinforcement, the inclined shear cracks extend to

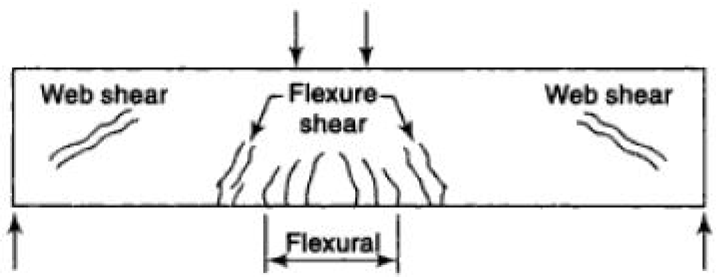


Fig. 8.2 *Types of Shear Cracks in Structural Concrete Members*

the compression face resulting in sudden explosive failures. This is sometimes referred to as the diagonal tensile mode of failure.

8.2.2 Failure due to Web-Shear Cracks

The ultimate shear resistance of prestressed concrete sections with web-shear cracking but without flexural cracks, is mainly governed by the limiting value of the principal tensile stress developed in concrete. The failure is assumed to take place when the principal tension exceeds the tensile strength of the concrete.

If V_{cw} = ultimate shear resistance of concrete in a section due to web-shear cracks

b_w = breadth of web of a member

h = overall depth of a member

f_{cp} = compressive prestress at the centroid of a section

f_t = tensile strength of concrete

S = statical moment (first moment of area)

I = second moment of area

τ_v = maximum shear stress at failure

For the condition at failure, equating the minor principal stress with the tensile strength of concrete, we get

$$\left[\left(\frac{f_{cp}}{2}\right) - 1/2\sqrt{f_{cp}^2 + 4\tau_v^2}\right] = -f_t$$

$$\left[\left(\frac{f_{cp}}{2}\right) - \frac{1}{2}\sqrt{f_{cp}^2 + 4\left(\frac{V_{cw} S}{Ib}\right)^2}\right] = -f_t$$

On simplification

$$V_{cw} = b_w \left(\frac{I}{S}\right)\sqrt{f_t^2 + f_{cp} f_t}$$

The value of (I/S) varies from 0.67 h (for rectangular sections) to 0.85 h (for flanged sections).

The British code (BS: 8110–1985)[6] and the Indian standard code (IS: 1343–1980)[7] specify a modified version of this relation given by,

$$V_{cw} = 0.67 b_w\, h\sqrt{(f_t^2 + 0.8 f_{cp} f_t)} \tag{8.1}$$

in which the value of 0.67 h is somewhat lower for flanged sections. This, together with reduced value of 0.8 f_{cp}, results in conservative estimates of the shear resistance of flanged sections. If there are inclined cables, the shearing force V_{cw} is increased by an amount equal to the vertical component of the prestressing force. In the above expression for computing V_{cw}, the tensile strength of concrete may be assumed as

$$f_t = 0.24\sqrt{f_{ck}}$$

8.2.3 Failure due to Flexure-Shear Cracks

The recommendations of both the British and Indian standard codes are similar for the computation of the ultimate shear resistance V_{cf} of sections cracked in flexure, which is expressed as

$$V_{cf} = \left(1 - 0.55\frac{f_{pe}}{f_p}\right)\tau_c b_w d + \left(\frac{M_0}{M}\right)V \tag{8.3}$$

$$\not< 0.1 b_w d\sqrt{f_{ck}}$$

where f_{pe} = effective prestress after all losses which shall not be put greater than $0.6 f_p$

f_p = characteristic strength of prestressing steel

τ_c = ultimate shear stress capacity of concrete compiled in Table 8.1

b_w = breadth of the member, which for flanged sections shall be taken as the breadth of the web

d = effective depth to tendons

M_0 = moment necessary to produce zero stress in the concrete at the extreme

V and M = the shear force and bending moment respectively at the section

8.3 DESIGN OF SHEAR REINFORCEMENTS

8.3.1 Indian Code (IS: 1343–1980) Recommendations

At any given section, the ultimate shear resistance V_c of the concrete alone should be taken as the lesser of the values of V_{cw} and V_{cf}. When V, the shear force due to the ultimate loads, is less than V_c, the shear force which can be carried by the concrete, minimum shear reinforcement should be provided in the form of stirrups such that,

$$S_v = \left(\frac{A_{sv}\,0.87 f_y}{0.4b}\right)$$

where S_{cv} = spacing of stirrups along the length of a member
A_{sv} = total cross-sectional area of stirrups legs effective in shear,
b = breadth, of the member which for T, I and L beams should be taken as the breadth of the rib b_w
f_y = characteristic strength of the stirrup reinforcement which shall not be taken greater than 415 N/mm^2

If the shear force V is less than 0.5 V_c and in a member of minor importance, shear reinforcement need not be provided.

When V exceeds V_c, shear reinforcement is required conforming to the relation,

$$S_v = \left(\frac{A_{sv}\,0.87 f_y d_t}{V - V_c}\right) \tag{8.4}$$

where d_t = depth from the extreme compression fibre either to the longitudial bars or to the centroid of tendons, whichever is greater.

Table 8.2 *Maximum Shear Stress (N/mm^2) (IS: 1343–1980)*

Concrete grade	*M–30*	*M–35*	*M–40*	*M–45*	*M–50*	*M–55 and above*
Maximum shear stress (N/mm^2)	3.5	3.7	4.0	4.3	4.6	4.8

The spacing of stirrups should exceed neither 0.75 d_t nor 4 times the web thickness for flanged members. When V exceeds 1.8 V_c, the maximum spacing should be reduced to 0.5 d_t. The lateral spacing of the individual legs of the stirrups provided at a cross-section should not exceed 0.75 d_t.

The maximum shear stress permissible for different grades of concrete are shown in Table 8.2. If the nominal shear stress (V/bd) exceeds these values, the section has to be redesigned.

EXAMPLE 8.10 The support section of a prestressed concrete beam, 100 mm wide and 250 mm deep, is required to support an ultimate shear force of 60 kN. The compressive prestress at the centroidal axis is 5 N/mm^2. The characteristic cube strength of concrete is 40 N/mm^2. The cover to the tension reinforcement is 50 mm. If the characteristic tensile strength of steel in stirrups is 250 N/mm^2, design suitable reinforcements at the section using the Indian standard code IS: 1343 recommendations.

Given data:

$b_w = 100$ mm $\qquad f_{cp} = 5$ N/mm^2

$h = 250$ mm $\qquad f_{ck} = 40$ N/mm^2

$d = 200$ mm $\qquad f_y = 250$ N/mm^2

$V = 60$ kN

For the support section uncracked in flexure,

$$V_c = V_{cw} = 0.67\, b_w h \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517 \text{ N/mm}^2$$

$$V_c = 0.67 \times 100 \times 250 \sqrt{1.517^2 + (0.8 \times 5 \times 1.517}$$

$$= 48457 \text{ N} = 48.4 \text{ kN}$$

$\therefore$ Balance shear $= (V - V_c) = (60 - 48.4)$

$= 11.6$ kN

Using 6 mm diameter two-legged stirrups, the spacing is obtained as

$$S_v = \left[\frac{A_{sv}\, 0.87 f_y d}{(V - V_c)}\right] = \left[\frac{2 \times 28.2 \times 0.87 \times 250 \times 200}{11600}\right] = 211.5 \text{ mm}$$

Maximum permissible spacing $= 0.75d = (0.75 \times 200) = 150$ mm

$\therefore$ Adopt 6 mm diameter two-legged stirrups at 150 mm centres.

Pre stressed concrete beams behavior in torsion:

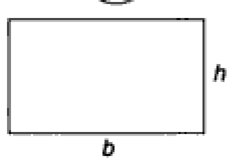
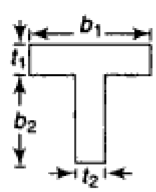
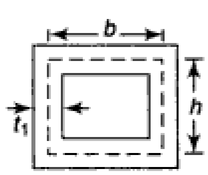
8.4 PRESTRESSED CONCRETE MEMBERS IN TORSION

8.4.1 Shear and Principal Stresses due to Torsion

In the case of structural concrete members subjected to torsion, shear streesses develop depending upon the type of cross section and magnitude of torque. The shear stresses in association with the flexural stresses may give rise to principal tensile stress, the value of which when it exceeds tensile strength of the concrete results in the development of cracks on the surface of the member.

The distribution of torsional shear-stress is uniform in circular sections where the magnitude of the shear stress is proportional to the distance from the centre. In the case of non circular sections involving warping of the cross-section, approximate formulae have been proposed based on the elastic analysis, due to St. Venant[12] and Bach[13], to estimate the maximum torsional shear stress for uncracked elements. The values suggested by Seely and Smith[14] for different cross-sections are compiled in Table 8.3. An analysis of principal stresses in prestressed concrete members should include the combined effect of shear stress due to transverse loads and torsion, together with direct stresses due to flexure and prestress.

Table 8.3 *Shear Stress in Members due to Torsion*

Sl. No.	*Name of section*	*Shape of section*	*Maximum shear stress*
1.	Circle		$(16\ T/\pi D^3)$
2.	Rectangle		$(T/\alpha\, b\, h^2)$ where α varies from 0.208 to 0.333 as (b/h) varies from 1 to ∞ (Refer Appendix-5)
3.	Flanged sections		$(3\ T t_i)/(\Sigma b_i t_i^3)$ where t_i is t_1 or t_2 and b_i is b_1 or b_2
4.	Box sections		$(T/2\ A\ t_i)$ where, $A = bh$

EXAMPLE 8.11 A pretensioned girder having a T-section is made up of a flange 200 mm wide and 60 mm thick. The overall depth of the girder is 660 mm. The thickness of the web is 60 mm. The horizontal prestress at a point 300 mm from the soffit is 10 N/mm^2. The shear stress due to transverse load acting at the same point is 2.5 N/mm^2. Determine the increase in the principal tensile stress at this point if the T-section is subjected to a torque of 2 kN m.

Principal tensile stress (without torque) is given by

$$f_{min} = \left(\frac{10}{2}\right) - \frac{1}{2}\sqrt{10^2 + 4 \times 2.5^2} = -0.6 \text{ N/mm}^2 \text{ (tension)}$$

Shear stress due to torque at the centre of the web,

$$\tau_t = \left[\frac{3Tt_i}{\Sigma b_i t_i^3}\right] = \left[\frac{3 \times 2 \times 10^6 \times 60}{(60^3 \times 200) + (60^3 \times 600)}\right] = 2.1 \text{ N/mm}^2$$

$\therefore$ Total shear stress = (2.5 + 2.1) = 4.6 N/mm^2

Principal tensile stress (with torque) is given by,

$$f_{min} = \left(\frac{10}{2}\right) - \frac{1}{2}\sqrt{10^2 + 4 \times 4.8^2} = -1.8 \text{ N/mm}^2 \text{ (tension)}$$

Increase in principal tensile stress due to torque is

$$= (1.8 - 0.6) = 1.2 \text{ N/mm}^2$$