UNIT-IV

Design for flexural resistance:

Types of flexural failure

1. Fracture of steel:- A minimum longitudinal reinforcement of 0.2% of the total concrete area shall be provided in all the cases except in the case of pre stressed units of small sections. This reinforcement may be further reduced to 0.15% in the case of HYSD bars. The percentage of steel provided, both tensioned & un-tensioned taken together should be sufficient so that when the concrete in pre compressed tensile zone cracks, the steel is in position to take up the additional tensile stress, transferred on to it by the cracking of the adjacent fiber of concrete & a sudden failure is avoided.

2. Failure of over reinforced section:- When the effective reinforcement index , which is expressed in terms of the percentage of reinforcement, the compressive strength of the concrete and the tensile strength of the steel, exceeds a certain range of values, the section is said to be over reinforced. Generally, over-reinforced members fall by the sudden crushing of concrete, the failure being characterized by small deflection and narrow cracks. The area of steel being comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength & in many cases it may well be within the proof stress of the tendon.

3. Failure of under reinforced section:- If the cross-section is provided with a steel greater than the minimum prescribed above, the failure is characterized by an excessive elongation of steel followed by crushing of concrete. This type of behaviors is generally desirable since there is considerable warning before the impending failure. As such, it is common practice to design the under-reinforced sections, which become more important in case of statically indeterminate structure.

Design methods:

Strain compatibility method

Assumptions:

1. The stress distribution in the compression zone of concrete can be defined by means of coefficients applied to the characteristic compressive strength & the average compressive stress & the position of the centre of compression can be assessed.

2. The distribution of concrete strain is linear. (i.e. the plane section normal to axis remains plane after bending)
3. The resistance of concrete in tension is neglected.
4. The maximum compressive strain in concrete at failure reaches at a particular level.

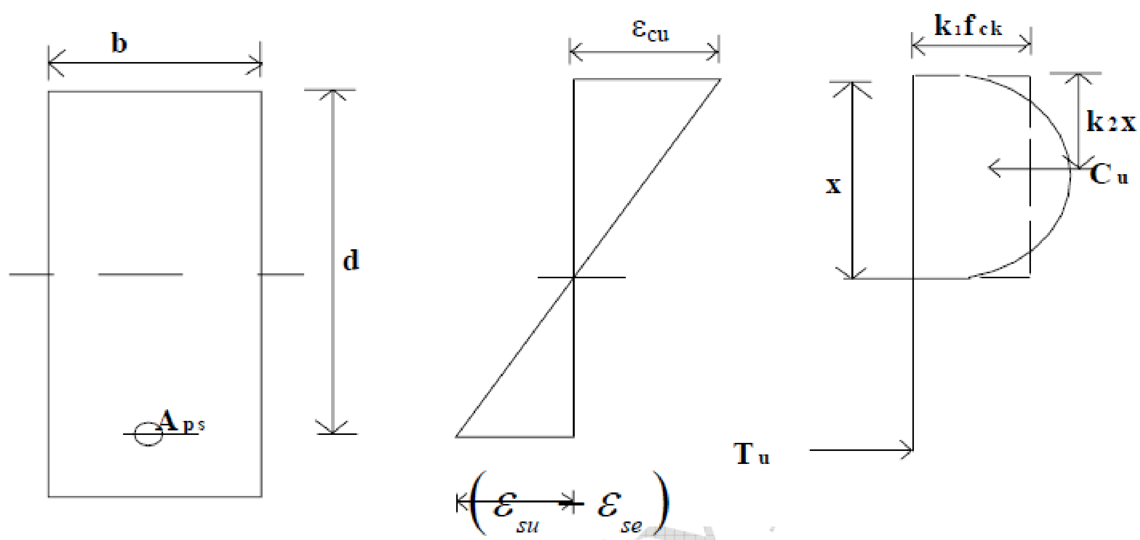


Fig. Stress-Strain Distribution at Failure

Total compressive force, $C_u = k_1 f_{ck} bx$

Total tensile force, $T_u = A_{ps} f_{pb}$

The ultimate flexural strength of the concrete, $M_u = k_1 f_{ck} bx\left(d - K_2 x\right)$

Where,

f_{ck} →Characteristic strength of the prestressing concrete

b→Effective width

x →Neutral axis depth

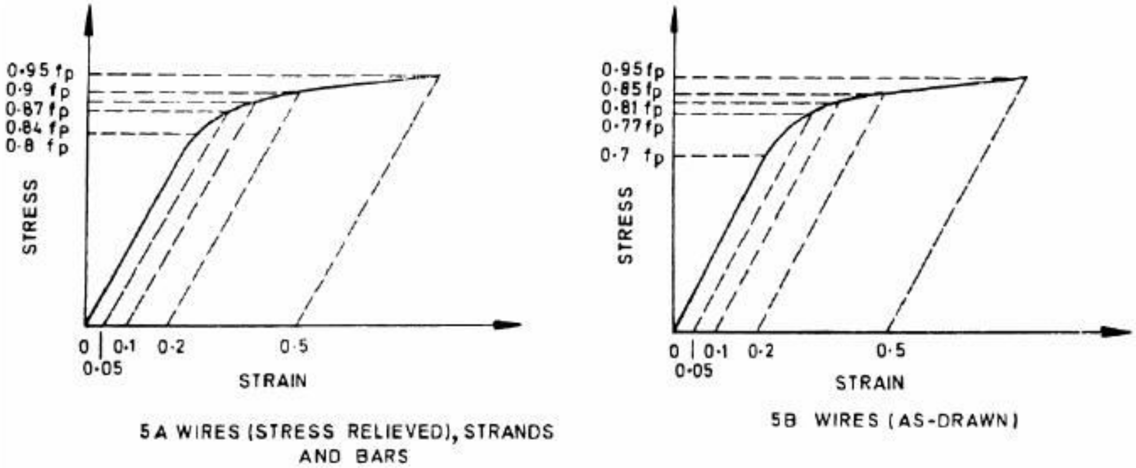
A_{ps} →Area of prestressing tendons

f_{pb} →Characteristic tensile strength of the prestressing tendon

k_1 →A constant whose value varies from 0.5-0.7 for f_{ck} = 60-20 Mpa

k_2 →A constant whose value varies from 0.42-0.47 for f_{ck} = 60-20 Mpa

The typical stress strain characteristic of different types of tendons used in prestressed concrete as recommended by IS1343 is given in the figure below.



REPRESENTATIVE STRESS STRAIN CURVES FOR PRESTRESSING STEELS

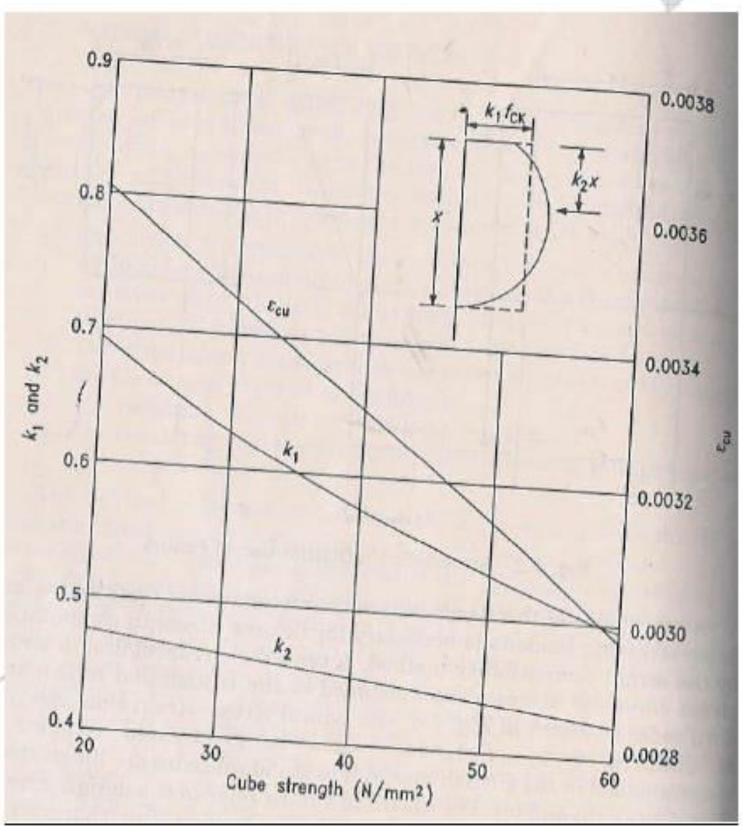


Fig. Characteristic of Hognestad et al's stress block

<u>Steps</u>

1. Compute the effective strain ε_{se} in steel due to prestress after allowing for all losses from the stress-strain curve of steel.

2. Assume a trial value for neutral axis depth x & evaluate $\left(\varepsilon_{su} - \varepsilon_{se}\right)$
3. Using the stress-strain curve for steel, determine the value of stress in steel at failure f_{pb} corresponding to ε_{su}
4. Compute total compression C & tension T.
5. If $C = T$ the assumed value of x is OK. Otherwise if tension is less than the compression decrease the value of x & vice versa to repeat the process 2-4 again.
6. Evaluate moment of resistance (ultimate moment), $M = A_{ps} f_{pb}\left(d - k_2 x\right)$

Example

A pretensioned concrete beam with a rectangular section, 100 mm wide by 160 mm deep, is prestressed by 10 high–tensile wire of 2.5 mm diameter located at an eccentricity of 40 mm. The initial force in each wire is 6.8 kN. The strain loss in wires due to elastic shortening, creep and shrinkage of concrete is estimated to be 0.0012 units. The characteristic cube strength of the concrete is 40 N/mm^2. Given the load-strain curve of 2.5 mm diameter steel wire (fig below), estimate the ultimate flexural strength of the section using strain compatibility method.

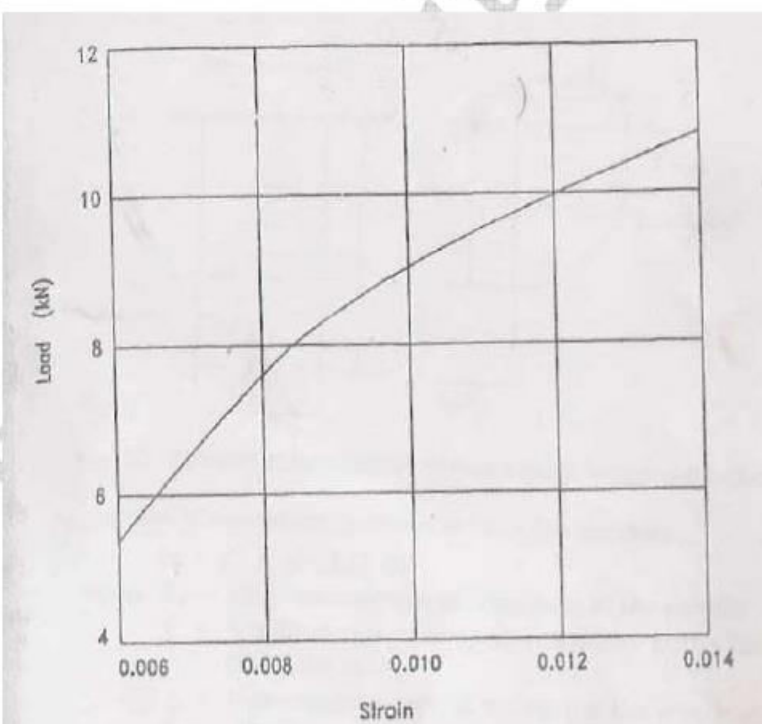


Fig. 7.6 Load-Strain Curve for 2.5 mm Wire

Solution:-

From the above figure (Hognestad stress block) we can have for f_{ck}= 40 N/mm^2,

ε_{cu} = 0.0033 ,;k$_1$ = 0.57 ; k$_2$ = 0.45.

As per figure strain due to load of 6.8 kN in wire is 0.0073

So effective strain in steel after all losses is given by

$\varepsilon_{se} = (0.0073 - 0.0012) = 0.0061$

First trial

Assuming x = 60 mm

From the strain diagram $(\varepsilon_{su} - \varepsilon_{se}) = 0.0033$

Therefore, $\varepsilon_{su} = (0.0033 + 0.0061) = 0.0094$

Corresponding force in the wire =8.4 kN

Total tensile force = 10 × 8.4 = 84 kN.

Total compressive force = $(k_1 \cdot f_{ck} \cdot b \cdot x)$

$$= \frac{(0.57 \times 40 \times 100 \times 60)}{1000} = 136.8 \text{ kN}$$

Since tension is less than compression, x is decreased for second trial

Second trial

_Assuming x = 43 mm

From the strain diagram $(\varepsilon_{su} - \varepsilon_{se}) = 0.0059$

$\varepsilon_{su} = (0.0059 + 0.0061) = 0.012$

Corresponding force in the wire =9.9kN

Total tensile force $= (10 \times 9.9) = 99$ kN

Total compressive force $= \frac{(0.57 \times 40 \times 100 \times 43)}{1000} = 98$ kN

Since tension is nearly equal to compression, strain compatibility is established.

$$M_u = A_{ps} f_{pb} (d - k_2 x)$$

$$= 99 \times 10^3 (120 - 0.45 \times 43)$$

$$= 9.96 \times 10^6 \text{ Nmm}$$

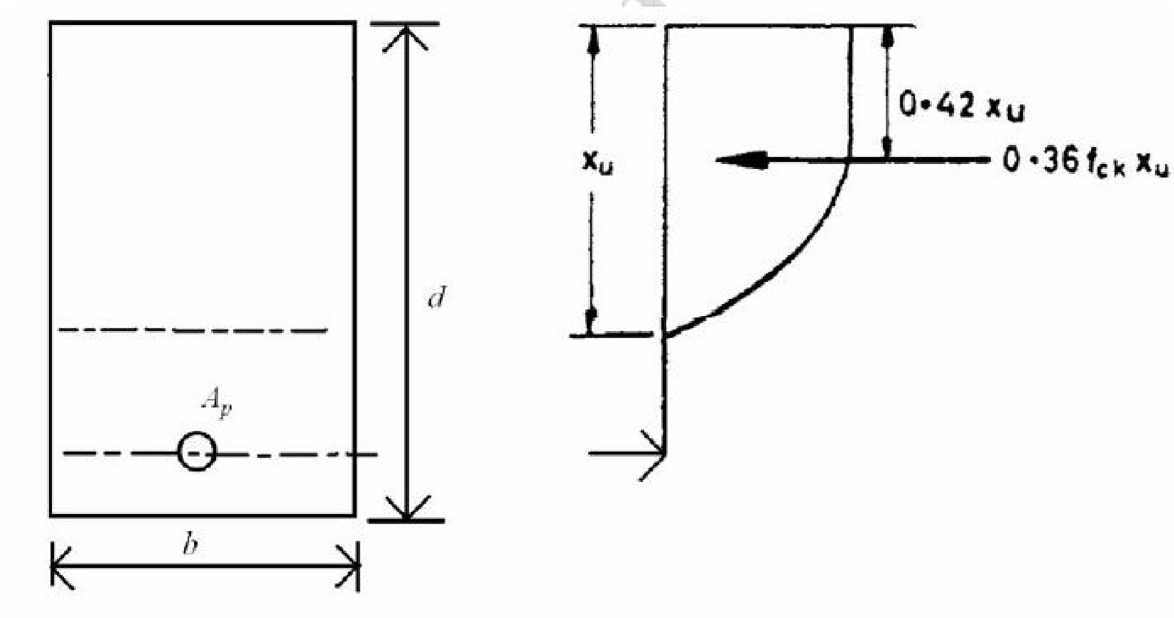
$$= 9.96 \text{ kNm}$$

Indian code (as per IS 1343 1980)

<u>Assumption</u>

1. The plane sections normal to the axis remain plane after bending.
2. The maximum strain in concrete at outermost compression fibre is taken as 0.35% in bending regardless of the strength of concrete.
3. The relationship between stress & strain distribution in concrete is assumed to be parabolic. The maximum compressive stress is equal to $0.446 f_{ck}$

   Where, f_{ck} → Characteristic strength of the concrete
4. The tensile strength of concrete is ignored.
5. The steel & concrete are bonded completely.
6. The stresses in bonded prestressing tendons are derived from the respective stress-strain curve for the particular steel.



If

M_u → Ultimate moment of resistance of the section

f_{pu} → Tensile strength developed in tendons at the failure stage of the beam

f_p → Characteristic tensile strength of the prestressing steel

f_{pe} → Effective prestress in tendon after losses

A_p →Area of prestressing tendons

d →Effective depth

b →Effective width

x_u →Neutral axis depth

Then moment of resistance, $M_u = f_{pu} A_p \left(d - 0.42 x_u\right)$

The value of f_{pu} depends upon the effective reinforcement ratio

$$\left(\frac{A_p f_p}{b d f_{ck}}\right)$$

For pre-tensioned & post-tensioned members with an effective bond between concrete & tendon, the value of f_{pu} & x_u are given in table 11 of IS1343. The effective prestress f_{pe} after all losses should not be less than 0.45 f_p . For post-tensioned rectangular beams with un-bonded tendons, the value of f_{pu} & x_u are influenced by the effective span to depth ratios, and their values for different span/depth ratios are given in Table-12 of IS1343.

TABLE 11 CONDITIONS AT THE ULTIMATE LIMIT STATE FOR RECTANGULAR BEAMS WITH PRE-TENSIONED TENDONS OR WITH POST-TENSIONED TENDONS HAVING EFFECTIVE BOND

$\frac{A_p f_p}{b d f_{ck}}$	STRESS IN TENSION AS A PROPORTION OF THE DESIGN STRENGTH $\frac{f_{pu}}{0.87 f_p}$		RATIO OF THE DEPTH OF NEUTRAL AXIS TO THAT OF THE CENTROID OF THE TENDON IN THE TENSION ZONE x_u/d	
	Pre-tensioning	Post-tensioning with effective bond	Pre-tensioning	Post-tensioning with effective bond
(1)	(2)	(3)	(4)	(5)
0.025	1.0	1.0	0.054	0.054
0.05	1.0	1.0	0.109	0.109
0.10	1.0	1.0	0.217	0.217
0.15	1.0	1.0	0.326	0.316
0.20	1.0	0.95	0.435	0414
0.25	1.0	0.90	0.542	0.488
0.30	1.0	0.85	0.655	0.558
0.40	0.9	0.75	0.783	0.653

TABLE 12 CONDITIONS AT THE ULTIMATE LIMIT STATE FOR POST-TENSIONED RECTANGULAR BEAMS HAVING UNBONDED TENDONS

(*Clause* B-1)

$\frac{A_p f_p}{bd f_{ck}}$	STRESS IN TENDONS AS A PROPORTION OF THE EFFECTIVE PRESTRESS f_{pu}/f_p FOR VALUES OF l/d $\left(\frac{\text{EFFECTIVE SPAN}}{\text{EFFECTIVE DEPTH}}\right)$			RATIO OF DEPTH OF NEUTRAL AXIS TO THAT OF THE CENTROID OF THE TENDONS IN THE TENSION ZONE x_u/d FOR VALUES OF l/d $\left(\frac{\text{EFFECTIVE SPAN}}{\text{EFFECTIVE DEPTH}}\right)$		
	30	20	10	30	20	10
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.025	1.23	1.34	1.45	0.10	0.10	0.10
0.05	1.21	1.32	1.45	0.16	0.16	0.18
0.10	1.18	1.26	1.45	0.30	0.32	0.36
0.15	1.14	1.20	1.36	0.44	0.46	0.52
0.20	1.11	1.16	1.27	0.56	0.58	0.64

Moment of resistance of flanged section

The ultimate moment of resistance of flanged sections in which the neutral axis falls outside the flange is computed by combining the moment of resistance of web & flange portion & considering the stress block is shown below.

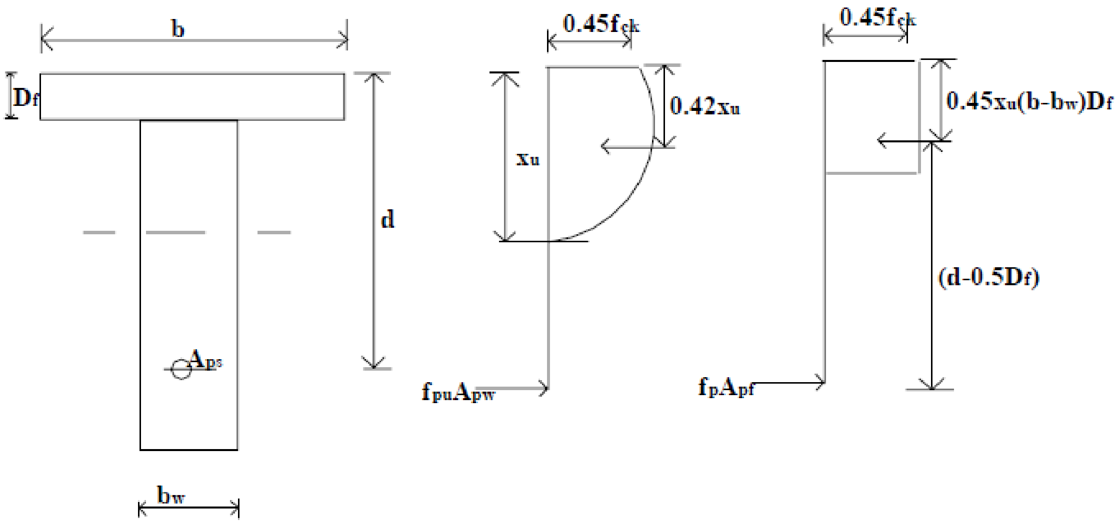


Fig. Moment of Resistance of flanged sections (x_u>0.5D_f)

If A_{pw} → Area of prestressing steel for web

A_{pf} → Area of prestressing steel for flange

D_f → Thickness of flange

Then, $A_p = \left(A_{pw} + A_{pf}\right)$

But, $A_{pf} = 0.45 f_{ck} \left(b - b_w\right)\left(\dfrac{D_f}{f_p}\right)$

After evaluating A_{pf}, the value of A_{pw} is obtained as

$$A_{pw} = \left(A_p - A_{pf}\right)$$

For effective reinforcement ratio of $\left(\dfrac{A_{pw} f_p}{b_w d f_{ck}}\right)$ the corresponding values of $\left(f_{pu}/0.87 f_p\right)$ & $\left(\dfrac{x_u}{d}\right)$ are obtained from Table – 11 of IS 1343 1980. The moment of resistance of the flange section is obtained from the expression

$$M_u = f_{pu} A_{pw}\left(d - 0.42 x_u\right) + 0.45 f_{ck}\left(b - b_w\right) D_f \left(d - 0.5 D_f\right)$$

Example

A pretensioned prestressed concrete beam having a rectangular section 150 mm wide 350 mm deep has an effective cover of 50 mm If f_{ck} = 40 N/mm^2 & f_p = 1600 N/mm^2 & the area of prestressing steel A_p = 461 mm^2, calculate the ultimate flexural strength of the section using IS code method.

Solution:-

Given data,

Characteristic strength of concrete, f_{ck}=40 N/mm^2

Characteristic strength of tendon, f_p = 1600 N/mm^2

Area of tendon, A_p =461 mm^2;

Width, b = 150 mm

Effective depth, d = 300 mm

The effective reinforcement ratio is given by

$$\left(\frac{f_p A_p}{f_{ck} bd}\right) = \left(\frac{1600 \times 461}{40 \times 150 \times 300}\right) = 0.40$$

From Table- 11, the corresponding values of the ratios are

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = \ 0.9 \text{ and } \left(\frac{x_u}{d}\right) = 0.783$$

$$\therefore f_{pu} = (0.87 \times 0.9 \times 1600) = 1253 \text{ N/mm}^2$$

$$\therefore x_u = (0.783 \times 300) \ = 234.9 \text{ mm}$$

Hence, the ultimate flexural strength of the section is

$$M = f_{pu} A_p \left(d - 0.42 x_u\right)$$

$$= 1253 \times 461 (300 - 0.42 \times 234.9)$$

$$= 116 \times 10^6 \text{ Nmm } = 116 \text{ kNm}$$

Example

A pretensioned T-section has a flange, which is 300 mm wide 200 mm thick. The rib is 150 mm wide by 350 mm deep. The effective depth of the cross section is 500 mm. Given: A_p =200 mm^2, f_{ck}=50 N/mm^2 & f_p = 1600 N/mm^2, estimate the ultimate moment capacity of the T-section using IS code method.

Solution:-

Given data,

Characteristic strength of concrete, f_{ck} = 50 N/mm^2

Characteristic strength of tendon, f_p = 1600 N/mm^2

Area of tendon, A_p =200 mm^2;
Width, b = 300 mm; Depth, d = 500 mm

Assuming that the neutral axis falls within the flange, the value of b = 300 mm for computations of effective reinforcement ratio.

$$\therefore \left(\frac{f_p A_p}{f_{ck} bd}\right) = \left(\frac{1600 \times 200}{50 \times 300 \times 500}\right) = 0.04$$

From Table- 11, the corresponding values of the ratios are

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0 \text{ and } \left(\frac{x_u}{d}\right) = 0.09$$

$$\therefore f_{pu} = (0.87 \times 1600) = 1392 \text{ N/mm}^2$$

$$\therefore x_u = (0.09 \times 500) = 45 \text{ mm}$$

The assumption that the neutral axis falls within the flange is correct. Hence, the ultimate flexural strength of the section is:

$$\begin{aligned} M &= f_{pu} A_p (d - 0.42 x_u) \\ &= 1392 \times 200 (500 - 0.42 \times 45) \\ &= 134 \times 10^6 \text{ Nmm} = 134 \text{ kNm} \end{aligned}$$

Deflections

Factors influencing deflection:

1. Imposed load & self load
2. Magnitude of prestressing force
3. Cable profile
4. Second moment of area of cross-section
5. Modulus of elasticity of concrete
6. Shrinkage, creep & relaxation of steel stress
7. Span of the member
8. Fixity condition

Short-term deflection of uncracked members

Mohr's theorem θ

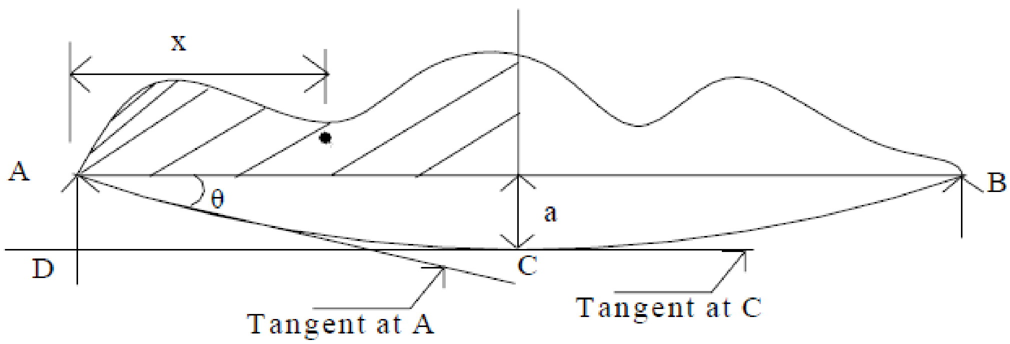


Fig. Slope and deflection of beam

If $\theta \rightarrow$ Slope of the elastic curve at A

$AD \rightarrow$ Intercept between the tangent at C & vertical at A

$a \rightarrow$ Deflection at the centre for symmetrically loaded simply supported beam

$A \rightarrow$ Area of the beam between A and C

$x \rightarrow$ Distance of the centroid of the BMD between A and C from the left support

$EI \rightarrow$ Flexural rigidity of the beam

Then by Mohr's first theorem

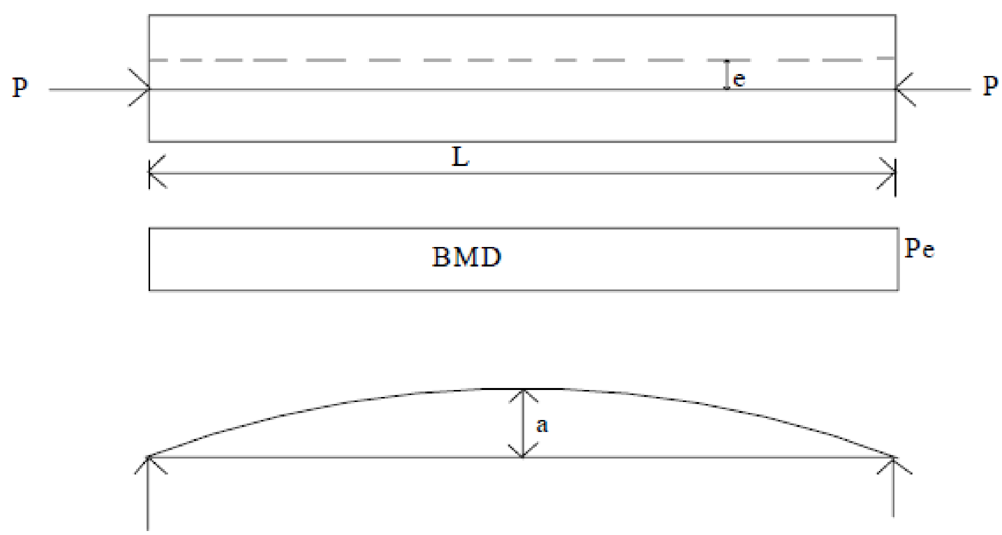
$$\text{Slope} = \frac{Area\ of\ BMD}{Flexural\ rigidity}\ ;\ \theta = \frac{A}{EI}$$

According to Mohr's second theorem

$$\text{Intercept, } a = \frac{moment\ \ of\ \ area\ \ of\ \ BMD}{Flexural\ \ rigidity} = \left(\frac{Ax}{EI}\right)$$

Effect of tendon profile on deflection

1. **Straight tendon**



If $P \rightarrow$ Effective prestressing force

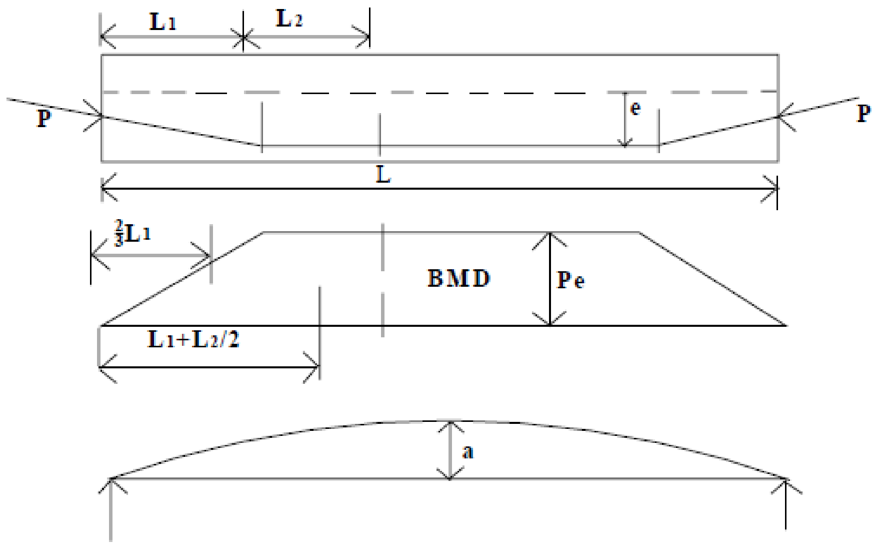
$e \rightarrow$ Eccentricity

$L \rightarrow$ Length of the beam

Then,

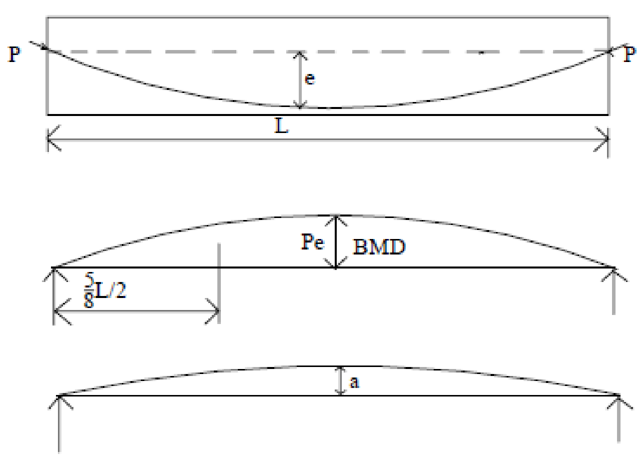
$$\text{Deflection, } a = -\frac{\left(Pe\frac{L}{2}\right)\left(\frac{L}{4}\right)}{EI} = -\frac{PeL^2}{8EI}$$

2. Trapezoidal tendon



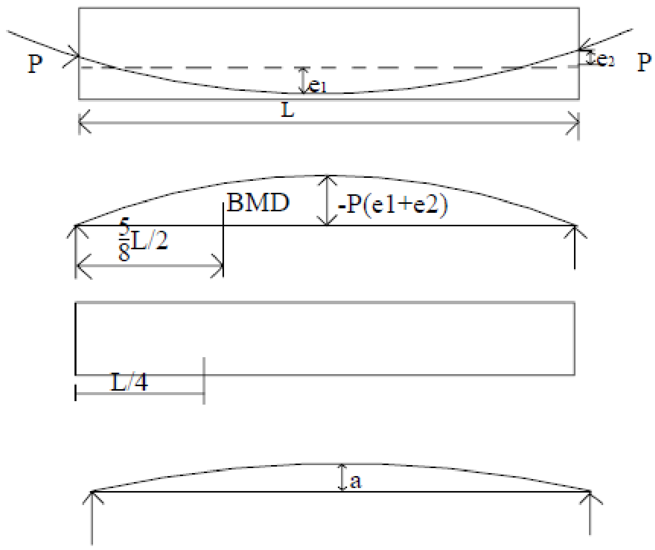
$$\text{Deflection, } a = \frac{Pe}{EI}\left[l_2\left(l_1 + l_2/2\right) + \left(\frac{l_1}{2}\right)\left(\frac{2}{3l_1}\right)\right] = -\frac{PeL^2}{8EI}\left(2l_1^2 + 6l_1l_2 + 3l_2^2\right)$$

3. Parabolic tendons (Central Anchors)



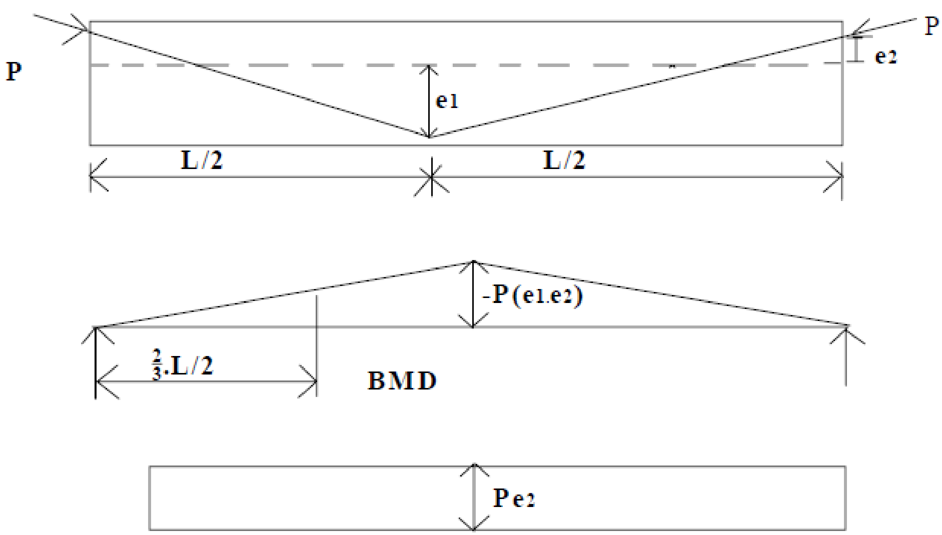
$$\text{Deflection, } a = \frac{Pe}{EI}\left[\frac{2}{3}\cdot\frac{L}{2}\cdot\frac{5}{8}\cdot\frac{L}{2}\right] = -\left(\frac{5PeL^2}{48EI}\right)$$

4. Parabolic tendons (Eccentric Anchors)



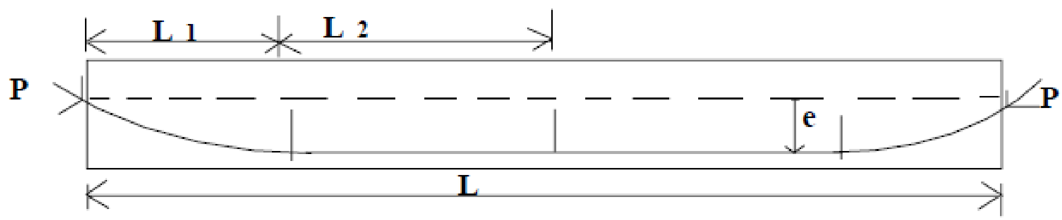
$$\text{Deflection, } a = \left[\frac{-5}{48}\frac{PL^2}{EI}(e_1+e_2)\right] + \left[\frac{Pe_2L^2}{8EI}\right] = \frac{PL^2}{48EI}(-5e_1+e_2)$$

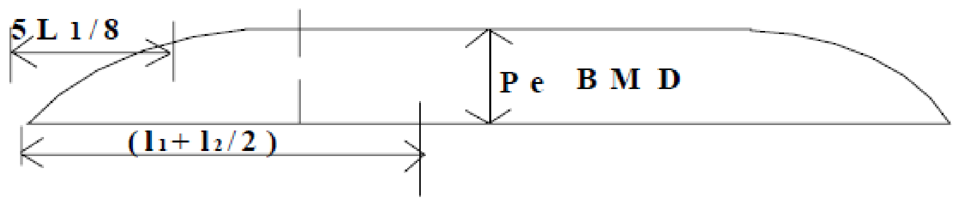
5. Sloping tendon (Eccentric Anchors)



$$\text{Deflection, } a = \left[-\frac{PL^2}{12EI}(e_1+e_2)\right] + \left[\frac{Pe_2L^2}{8EI}\right] = \frac{PL^2}{24EI}(-2e_1+e_2)$$

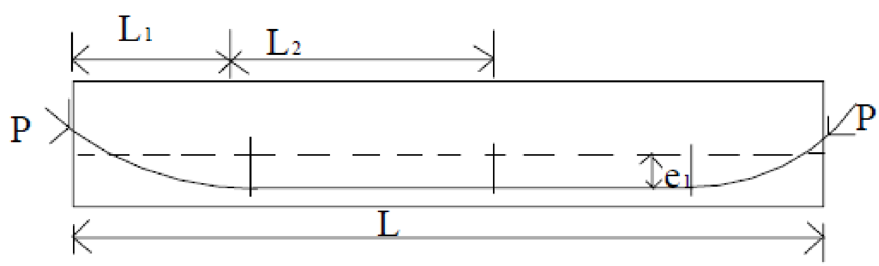
6. **Parabolic and straight Tendon**

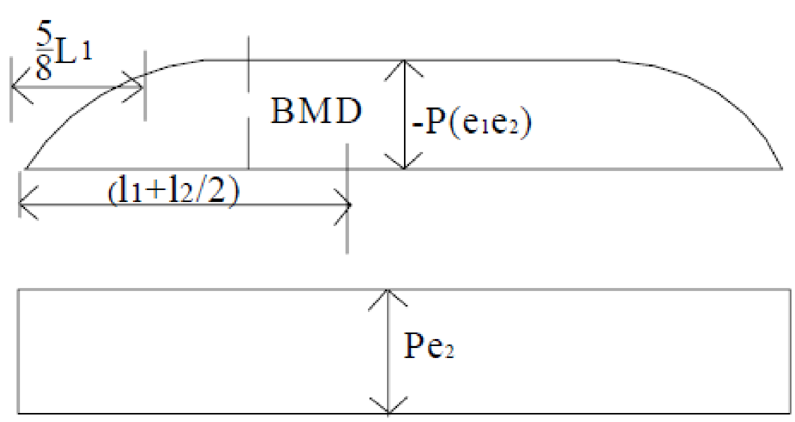




Deflection, $a = \frac{Pe}{EI}\left[\left(\frac{2l_1}{3}\right)\left(\frac{5l_1}{8}\right) + l_2\left(l_1 + \frac{l_2}{2}\right)\right] = -\frac{Pe}{12EI}\left[5l_1^2 + 12l_1l_2 + 6l_2^2\right]$

7. **Parabolic & straight tendons (Eccentric Anchors)**





Deflection, $a = -\frac{P(e_1+e_2)}{12EI}\left[5l_1^2 + 12l_1l_2 + 6l_2^2\right] + \left[\frac{Pe_2L^2}{8EI}\right]$

Load due to external loading

If $g \rightarrow$ Self weight of the beam per metre

$q \rightarrow$ Uniformly distributed superimposed load per metre

Then, Downward deflection, $a = \dfrac{5(g+q)L^4}{384EI}$

Example

A concrete beam with cross-sectional area of $32 \times 10^3 \text{mm}^2$ & the radius of gyration is 72mm is prestressed by a parabolic cable carrying an effective stress of 1000 N/mm^2.

The span of the beam is 8 m. The cable, composed of 6 wires of 7mm diameter, has an eccentricity of 50mm at the centre & zero at the supports. Neglecting all losses, find the central deflection of the beam as follows:

(a) self-weight + prestress

(b) self-weight + prestress + live load of 2 kN/m.

Solution: -

Data Provided;

Cross sectional area of beam, $A = 32 \times 10^3 \text{mm}^2$

Modulus of elasticity, E = 38 kN/mm^2; Dead weight of concrete, D_c = 24 kN/mm^3;

Radius of gyration , i = 72 mm; Span, L =8 m =8000 mm;

Eccentricity, e =50 mm

$I = Ai^2 = (32 \times 10^3 \times 72^2) = 166 \times 10^6 \text{mm}^4$

Prestressing force, $P = (6 \times 38.5 \times 1000) =$ 231000 N =231 kN

Self weight, $g = \left(\dfrac{32 \times 10^3}{10^6} \times 24\right)$ =0.77 kN/m =0.00077 kN/mm

Downward deflection due to self weight = $\left(\dfrac{5gL^4}{384EI}\right) = \left(\dfrac{5 \times 0.00077 \times 8000^4}{384 \times 38 \times 166 \times 10^6}\right)$ = 6.5 mm

Upward deflection due to prestressing force = $\left(\dfrac{5PeL^4}{48EI}\right) = \left(\dfrac{5 \times 231 \times 50 \times 8000^4}{48 \times 38 \times 166 \times 10^6}\right)$ =12.2 mm

Downward deflection due to live load = $\left(\dfrac{6.5}{0.77} \times 2\right)$ = 16.9 mm

(a) Deflection due to (self-weight + prestress) =(12.2 - 6.5) = 5.7 mm(↑)

(b) Deflection due to (self-weight + prestress + live load) = (6.5-12.2+16.9) =11.2 mm(↓)

EXAMPLE 6.3 A rectangular concrete beam of cross-section 150 mm wide and 300 mm deep is simply supported over a span of 8 m and is prestressed by means of a symmetric parabolic cable, at a distance of 75 mm from the bottom of the beam at mid span and 125 mm from the top of the beam at support sections. If the force in the cable is 350 kN and the modulus of elasticity of concrete is 38 kN/mm^2, calculate:

(a) the deflection at mid-span when the beam is supporting its own weight and
(b) the concentrated load which must be applied at mid-span to restore it to the level of supports.

$P = 350$ kN, $E_c = 38$ kN/mm^2,
$I = 3375 \times 10^5$ mm^4 $e_1 = 75$ mm, $e_2 = 25$ mm

Net deflection due to the prestressing force

$$= \frac{PL^2}{48EI}(-5e_1 + e_2) = \left(\frac{350 \times 8000^2}{48 \times 38 \times 3375 \times 10^5}\right)(-5 \times 75 + 25)$$

$$= 12.7 \text{ mm (upwards)}$$

Self-weight of the beam, $g = (0.15 \times 0.30 \times 24) = 1.08$ kN/m
$= 0.00108$ kN/mm

$$\text{Downward deflection due to self-weight} = \left(\frac{5 \times 0.00108 \times 8000^4}{384 \times 38 \times 3375 \times 10^5}\right) = 4.5 \text{ mm}$$

(a) Deflection due to (prestress + self-weight) $= (-12.7 + 4.5) = -8.2$ mm (upwards)
(b) If Q = concentrated load required at the centre of span,

Then, $$\left(\frac{QL^3}{48EI}\right) = 8.2$$

$$\therefore \quad Q = \left(\frac{8.2 \times 48 \times 38 \times 3375 \times 10^5}{8000^3}\right) = 9.9 \text{ kN}$$

Prediction of long time deflection:

The deformations of prestressed members change with time as a result of creep and shrinkage of concrete and relaxation of stress in steel. The deflection of prestressed members can be computed relative to a given datum, if the magnitude and longitudinal distribution of curvatures for the beam span are known for that instant based on the load history, which includes the prestressing forces and the live loads.

The prestressed concrete member develops deformations under the influence of two usually opposing effects, which are the prestress and transverse loads. The net curvature ϕ_t at a section at any given stage is obtained.

$$\phi_t = \phi_{mt} + \phi_{pt}$$

Prediction of long time deflection

$$\phi_t = \phi_{mt} + \phi_{pt}$$

$$\phi_{mt} = (1+\phi)\phi_i$$

Where, ϕ_{mt} → Change of curvature due to transverse load.

ϕ_{pt} → Change of curvature due to prestress

ϕ_t → Total curvature

ϕ → Creep coefficient

ϕ_i → Initial curvature immediately after the application of transverse load.

According to ACI committee,

$$\phi_{pt} = -\frac{Pie}{EI} + \frac{(P_i - P_t)e}{EI} - \frac{(P_i + P_t)e\phi}{2EI}$$

$$\Rightarrow \phi_{pt} = -\frac{Pie}{EI}\left[1 - \frac{L_p}{P_i} + \left(1 - \frac{L_p}{2P_i}\right)\phi\right]$$

Where, P_i → Initial prestress

P_t → Prestress after time t

e → *Eccentricity* of the prestressing force at the section

EI → Flexural rigidity

$L_p = (P_i - P_t)$ → Shrinkage and creep

Thus the total deflection after time t obtained from the above expression is

$$a_f = a_{it}(1+\phi) - a_{ip}\left[1 - \frac{L_p}{P_i} + \left(1 - \frac{L_p}{2P_i}\right)\phi\right]$$

Where, a_{it} → Initial deflection due to transverse load

a_{ip} → Initial deflection due to prestress

a_f → Final deflection after time t

Simplified method

$$a_f = \left[a_{it} - a_{ip}\frac{P_t}{P_i}\right](1+\phi)$$

Here ,

+ve(positive) sign refers upward deflection.

-ve(negative) sign refers downward deflection.

Example:

A concrete beam having a rectangular section 100 mm wide and 300 mm deep is prestressed by a parabolic cable carrying an initial force of 240 kN. The cable has an eccentricity of 50 mm at the centre of the span is concentric at the supports. If the span of the beam is 10m and live load is 2 kN/m. Estimate the long time deflection after 6 months if E = 38 kN/mm^2 & creep coefficient $\phi = 2.0$, loss of prestress = 20% of the initial stress after 6 months. Estimate the long time deflection at the center of the span at the stage assuming that the DL & LL are simultaneously applied after the release of prestress.

Solution:-

Here, Given

P_i = 240 kN.

$I = 225 \times 10^6$

e = 50 mm

$\phi = 2.0$

DL = $0.1 \times 0.3 \times 24 = 0.72$kN/m

LL = 2 kN/m

Loss of prestress = 20% of P_i

Short time deflection:

Initial deflection due to prestress $= \left(\dfrac{5P_i eL^2}{48EI}\right) = \left(\dfrac{5 \times 240 \times 50 \times (10 \times 1000)^2}{48 \times 38 \times 225 \times 10^6}\right)$ =14.7 mm (↑)

Deflection due to self weight and live loads $= \left[\dfrac{5(g+q)L^4}{384EI}\right]$

$$= \left[\frac{5 \times (0.00072 \times +0.002)(10 \times 1000)^4}{384 \times 38 \times 225 \times 10^6}\right]$$

= 41.5 mm (↓)

Therefore, net deflection = 41.5 – 14.7 = 26.8 mm (↓).

Long time deflection:

The long time deflection, $a_f = a_{it}(1+\phi) - a_{ip}\left[1 - \dfrac{L_p}{P_i} + \left(1 - \dfrac{L_p}{2P_i}\right)\phi\right]$

$$= 41.5\ (1+2) - 14.7\ \left[1 - \frac{0.2P_i}{P_i} + \left(1 - \frac{0.2P_i}{2P_i}\right) \times 2\right]$$

= (124.5 - 38) = 86.5 mm(↓)

Using simplified formula

$$a_f = \left[a_{it} - a_{ip}\frac{P_t}{P_i}\right](1+\phi)$$

$$= (41.5 - 14.7 \times 0.8)(1 + 2) = 89.1 \text{ mm}(\downarrow)$$

EXAMPLE 6.5 A simply supported beam with a uniform section spanning over 6 m is post-tensioned by two cables, both of which have an eccentricity of 100 mm below the centroid of the section at mid-span. The first cable is parabolic and is anchored at an eccentricity of 100 mm above the centroid at each end, the second cable is straight and parallel to the line joining the supports. The cross-sectional area of each cable is 100 mm^2 and they carry an initial stress of 1200 N/mm^2. The concrete has a cross-section of 2×10^4 mm^2 and a radius of gyration of 120 mm.

The beam supports two concentrated loads of 20 kN each at the third points of the span, $E_c = 38$ kN/mm^2. Calculate using Lin's simplified method

(a) the instantaneous deflection at the centre of span: and
(b) the deflection at the centre of span after 2 years, assuming 20 per cent loss in prestress and the effective modulus of elasticity to be one-third of the short-term modulus of elasticity.

$A = 2 \times 10^4 \text{ mm}^2 \qquad i = 120 \text{ mm}$

$I = Ai^2 = (2 \times 10^4 \times 120^2) = 288 \times 10^6 \text{ mm}^4$

$P = 120 \text{ kN} \qquad e_1 = e_2 = 100 \text{ mm}$

$L = 6000 \text{ mm}$

Self-weight, $g = 0.00048$ kN/mm

Concentrated loads at third points of span, $Q = 20$ kN

(a) Downward deflection due to self-weight of the beam

$$= \left[\frac{5 \times 0.00048 \times 6000^4}{384 \times 38 \times 288 \times 10^6}\right] = 0.74 \text{ mm}$$

Downward deflection due to concentrated loads $= \left(\dfrac{23QL^3}{648EI}\right)$

$$= \left[\frac{23 \times 20 \times 6000^3}{648 \times 38 \times 288 \times 10^6}\right] = 14.10 \text{ mm}$$

Deflection due to prestressing force

Deflection due to the parabolic cable $= \left(\dfrac{PL^2}{48EI}\right)(-5e_1 + e_2)$

$$= \left(\frac{120 \times 6000^2}{48 \times 38 \times 288 \times 10^6}\right)(-5 \times 100 + 100) = -3.27 \text{ mm (upward)}$$

Deflection due to the straight cable $= \left(\dfrac{-PeL^2}{8EI}\right) = -\left(\dfrac{120\times100\times6000^2}{8\times38\times288\times10^6}\right)$

$= -4.92$ mm (upward)

Instantaneous deflection due to (prestress + self-weight + live loads)

$= (-3.27 - 4.92) + 0.74 + 14.10 = 6.65$ mm (downward)

(b) At the end of two years,

$$E_{ce} = \frac{E}{3} \text{ and loss of prestress} = 20\%$$

$\therefore$ Upward deflection $= 3\,[0.8\,(3.27 + 4.92)] = 19.65$ mm

Downward deflection $= 3\,(0.74 + 14.10) = 44.52$ mm

Net downward deflection $= (44.52 - 19.65) = 24.87$ mm

EXAMPLE 6.6 A prestressed concrete beam having a cross-sectional area (A) of 5×10^4 mm^2 is simply supported over a span of 10 m. It supports a uniformly distributed imposed load of 3 kN/m, half of which is non-permanent. The tendon follows a trapezoidal profile with an eccentricity of 100 mm within the middle-third of the span and varies linearly from the third-span points to zero at the supports. The area of tendons $A_p = 350$ mm^2 have effective prestress of 1290 N/mm^2 immediately after transfer. Using the following data, calculate

1. the short-term deflections, and
2. the long-term deflections.

Assume $I_g = 4.5\times10^8$ mm^4	Density of concrete = 23.6 kN/m^3
$E_c = 34$ kN/mm^2	Creep coefficient = 2
$A = 5\times10^4$ mm^2	Concrete shrinkage, $\varepsilon_{cs} = 450\times10^{-6}$
$E_s = 200$ kN/mm^2	Relaxation of steel stress = 10%

1. Short-term deflection

Initial prestressing force, $P = (350\times1290) = 4{,}51{,}500$ N

Self-weight of the beam $= \left(\dfrac{5\times10^4}{10^6}\times23.6\right) = 1.18$ kN/m

Non-permanent load $= 1.5$ kN/m

Permanent load = dead load + sustained live-load

$= (1.18 + 1.5) = 2.68$ kN/m

(i) Deflection due to the prestressing force Referring to Fig. 6.3 and substituting the value of L_1 and L_2 in the equation for deflection, we have

$$a_p = -\frac{Pe}{6EI}\,[2L_1^2 + 6L_1L_2 + 3L_2^2]$$

$L_1 = 3.333$ m and $L_2 = 1.666$ m and $e = 100$ mm. Thus,

$$a_p = \left(\frac{4{,}51{,}500\times100}{6\times34\times10^3\times4.5\times10^8}\right)[2\times3333^2 + 6(3333\times1666) + 3\times1666^2]$$

$= -31$ mm (upwards)

(ii) Deflection due to non-permanent load (live-load)

$$q = 1.5 \text{ kN/m}$$

$$a_q = \left(\frac{5qL^4}{384EI}\right) = \left(\frac{5 \times 1.5 \times (10 \times 10^3)^4}{384 \times 34 \times 10^3 \times 4.5 \times 10^8}\right) = 12.8 \text{ mm (downwards)}$$

(iii) Deflection due to permanent load (sustained load)

$$g = 2.68 \text{ kN/m}$$

$$a_g = \left(\frac{5gL^4}{384EI}\right) = \left[\frac{5 \times 2.68 \times (10 \times 10^3)^4}{384 \times 34 \times 10^3 \times 4.5 \times 10^8}\right] = 22.8 \text{ mm (downwards)}$$

(iv) Short-term deflections

(a) When the non-permanent load is acting, the short-term deflection is given by

$$a_s = (-31 + 12.8 + 22.8) = 4.6 \text{ mm (downwards)}$$

(b) When the non-permanent load is not acting, the short-term deflection is given by

$$a_s = (-31 + 22.8) = -8.2 \text{ mm (upwards)}$$

2. Long-term deflection Stress in concrete at the level of steel

$$f_c = \left(\frac{4,51,500}{5 \times 10^4}\right) + \left(\frac{4,51,500 \times 100 \times 100}{4.5 \times 10^8}\right) = 19 \text{ N/mm}^2$$

$$\alpha_e = \left(\frac{E_s}{E_c}\right) = \left(\frac{200}{34}\right)$$

$$= 5.88$$

(a) Loss due to Relaxation $= 10\% = 129 \text{ N/mm}^2$

(b) Loss due to shrinkage $= (450 \times 10^{-6} \times 200 \times 10^3) = 90 \text{ N/mm}^2$

(c) Loss due to creep $= (2 \times 5.88 \times 19) = 223 \text{ N/mm}^2$

Total loss $= 442 \text{ N/mm}^2$

P = Initial prestressing force = 4,51,500 N

δP = Loss of prestressing force = (442 × 350) = 1,54,700 N

$(P - \delta P)$ = Final prestressing force = (4,51,500 – 1,54,700) = 2,96,800 N

$$\text{Average prestressmg force} = \left[\frac{(4,51,500 + 2,96,800)}{2}\right] = 3,74,150 \text{ N}$$

(i) Long-term deflection due to prestress

a_p = (Deflection due to the initial prestressing force (P)) – [Deflection due to loss of prestressing force (δP)] + (Deflection due to the average prestressing force due to creep with $\phi = 2$)

$$= 31 - \left(\frac{1,54,700 \times 31}{4,51,500}\right) + \left(\frac{3,74,150 \times 31}{4,51,500}\right)2$$

$$= 31 - 10.6 + 51.4 = 72 \text{ mm (upwards)}$$

(ii) Long-term deflection due to permanent load

$$a_g = (1 + \phi) \text{ (short-term deflection)}$$

$$= (1 + 2)(22.8) = 68.4 \text{ mm (downwards)}$$

(iii) Long-term deflection due to non-permanent load

$a_q = 12.8$ mm (downwards)

The total long-term deflection is computed as,

(a) When the non permanent load is acting:
Mid-span deflection = (–72 + 68.4 + 12.8) = 9.2 mm (downwards)

(b) When the non-permanent load is not acting:
Mid-span deflection = (–72 + 68.4) = –3.6 mm (upwards)

EXAMPLE 6.9 A prestressed concrete beam of rectangular section, 120 mm wide and 300 mm deep, spans over 6 m. The beam is prestressed by a straight cable carrying an effective force of 180 kN at an eccentricity of 500 mm. If it supports an imposed load of 4 kN/m and the modulus of elasticity of concrete is 38 kN/mm^2, compute the deflection at the following stages and check whether they comply with the IS code specifications.

(a) upward deflection under (prestress + self-weight), and
(b) final downward deflection under (prestress + self-weight + imposed load) including the effects of creep and shrinkage. Assume the creep coefficient to be 1.80.

$P = 180$ kN	Self-weight, $g = 0.86$ N/mm
$e = 50$ mm	Imposed load, $q = 4$ N/mm
$I = 27 \times 10^7$ mm^4	$E_c = 38$ kN/mm^2
$L = 6000$ mm	

Deflection due to the prestressing force

$$= \left(\frac{PeL^2}{8EI}\right) = \left(\frac{180 \times 50 \times 6000^2}{8 \times 38 \times 27 \times 10^7}\right) = 4.0 \text{ mm (upward)}$$

Deflection due to the self-weight of the beam

$$= \left(\frac{5gL^4}{384\,EI}\right) = \left(\frac{5 \times 0.86 \times 6000^4}{384 \times 38 \times 10^3 \times 27 \times 10^7}\right) = 1.4 \text{ mm}$$

Deflection due to self-weight and live load

$$= \left[\frac{5(g+q)L^4}{384\,EI}\right] = \left[\frac{5 \times 4.86 \times 6000^4}{384 \times 38 \times 10^3 \times 27 \times 10^7}\right] = 8.0 \text{ mm}$$

(a) Deflection due to (prestress + self-weight) = (–4.0 + 1.4) = –2.6 mm
Permissible upward deflection as per draft IS: 1343

$$= \left(\frac{\text{span}}{300}\right) = \left(\frac{6000}{300}\right) = 20 \text{ mm}$$

Hence, the upward deflection is within permissible limits.

(b) Deflection due to (prestress + self-weight + live load) including effects of creep and shrinkage

$$= (-4.0 + 8.0)\,(1 + \phi) = (4.0)\,(1 + 1.8) = 11.2 \text{ mm}$$

Permissible downward deflection as per IS: 1343

$$= \left(\frac{\text{span}}{250}\right) = \left(\frac{6000}{250}\right) = 24 \text{ mm}$$

Hence, the final downward deflection is within permissible limits.