STRUCTURAL ANALYSIS – I

Syllabus:

UNIT – I

PROPPED CANTILEVERS: Analysis of propped cantilevers-shear force and Bending moment diagrams-Deflection of propped cantilevers.

UNIT – II

FIXED BEAMS – Introduction to statically indeterminate beams with U. D. load central point load, eccentric point load. Number of point loads, uniformly varying load, couple and combination of loads shear force and Bending moment diagrams-Deflection of fixed beams effect of sinking of support, effect of rotation of a support.

UNIT – III

CONTINUOUS BEAMS: Introduction-Clapeyron's theorem of three moments-Analysis of continuous beams with constant moment of inertia with one or both ends fixed-continuous beams with overhang, continuous beams with different moment of inertia for different spans-Effects of sinking of supports-shear force and Bending moment diagrams.

UNIT-IV

SLOPE-DEFLECTION METHOD: Introduction, derivation of slope deflection equation, application to continuous beams with and without settlement of supports.

UNIT – V

ENERGY THEOREMS: Introduction-Strain energy in linear elastic system, expression of strain energy due to axial load, bending moment and shear forces - Castigliano's first theorem-Deflections of simple beams and pin jointed trusses.

UNIT – VI

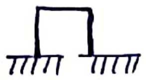
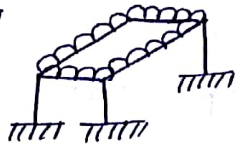
MOVING LOADS and INFLUENCE LINES: Introduction maximum SF and BM at a given section and absolute maximum S.F. and B.M due to single concentrated load U. D load longer than the span, U. D load shorter than the span, two point loads with fixed distance between them and several point loads-Equivalent uniformly distributed load-Focal length.

INFLUENCE LINES: Definition of influence line for SF, Influence line for BM- load position for maximum SF at a section-Load position for maximum BM at a sections, ingle point load, U.D. load longer than the span, U.D. load shorter than the span-Influence lines for forces in members of Pratt and Warren trusses.

UNIT-1

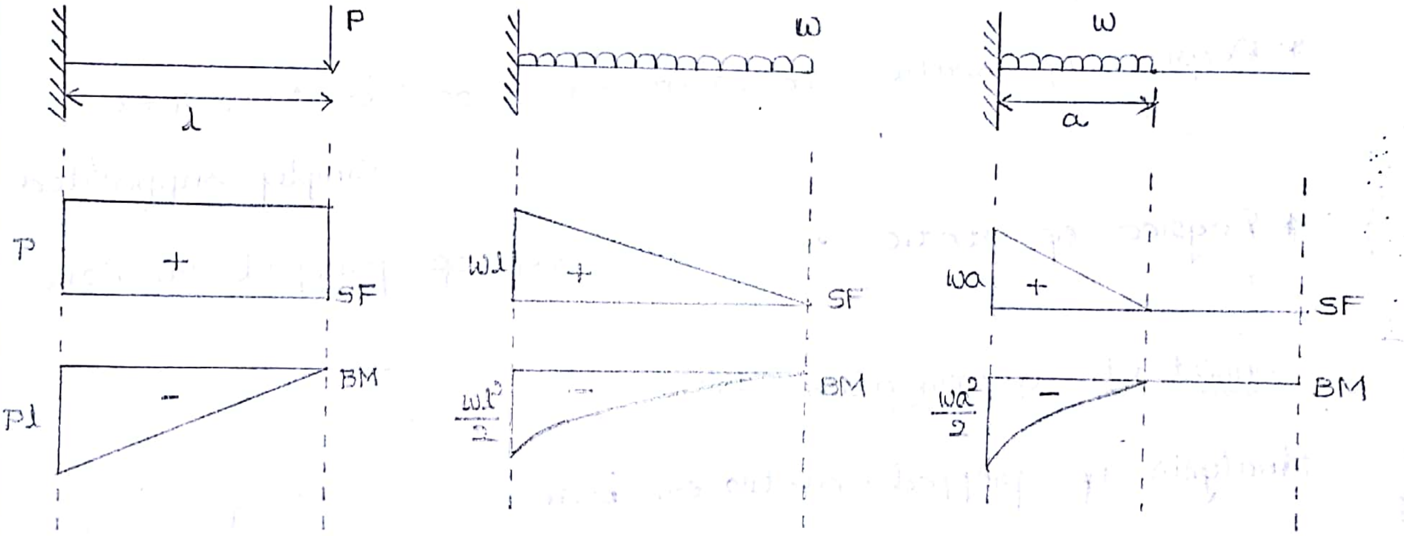
Propped - Cantilever

Structure: System of connected members to support the loads is called as structure.

* Plane structure 
* Space structure 

① Plane structure → In x direction of loads.

② Space structure → In x, y, z direction of loads.



Cantilever beam: When a beam fixed at one end free at another end then the beam is called as a cantilever beam.

Propped cantilever beam: When the beam fixed at one end and supported ~~at~~ any other ~~end~~ point on the beam, then the beam is called propped cantilever beam.

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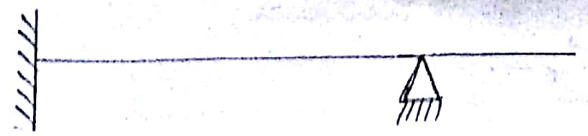
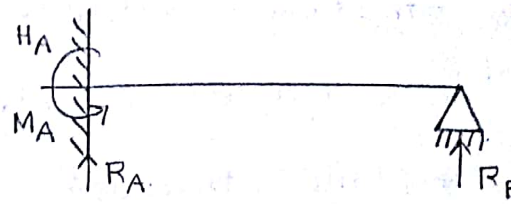


Degree of static indeterminancy:



$$D_s = r - 3$$
$$= 4 - 3$$
$$= 1$$

1 is the degree of static indeterminancy.

Definiton of degree of static indeterminancy:

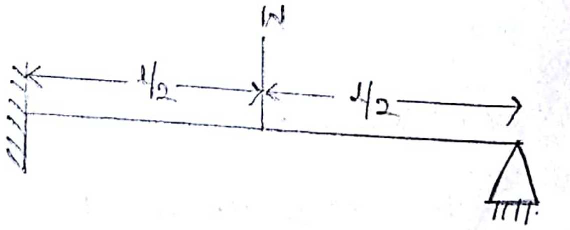
The no. of additional equatons required to field the reaction of the beam.

* Degree of static indeterminancy of cantilever = 0
simply supported = 0

* Degree of static indeterminancy of propped cantilever = 1

Consistent deformation Method:

Analysis of propped cantilever beam.



Analysing propped cantilever beam. as shown in figure.

Remove the prop applied on the beam and draw the bending moment diagram for the cantilever beam (fig 2) as showing in fig (3).

Find the deflection. at the prop by moment area. method.

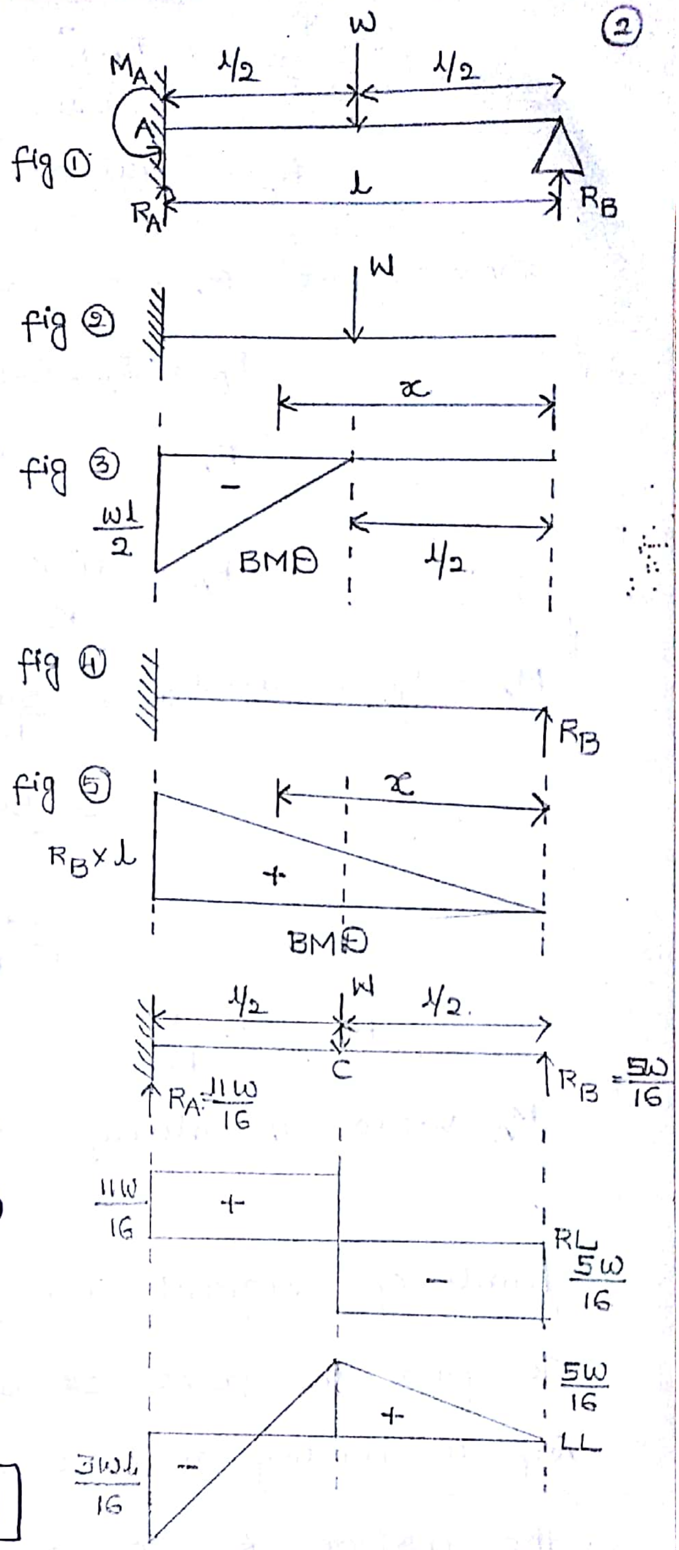
Let it be denoted as δB, due to loading

$$\delta B_1 = \frac{A\bar{x}}{EI}$$

$$= \frac{\left(\frac{1}{2} \times \frac{wl}{2} \times \frac{l}{2}\right)\left(\frac{l}{3} + \frac{l}{2}\right)}{EI}$$

$$= \frac{\frac{wl^2}{8}\left(\frac{2l+3l}{6}\right)}{EI}$$

$$\delta B_1 = \frac{5wl^3}{48EI}$$



Now remove the loading applied on the beam & draw the bending moment diagram for the prop. reaction let (R_B) as shown in fig 5.

and find the deflection at the prop. Due to prop reaction (R_B) Let it be denoted as (δB_2)

$$\delta B_2 = \frac{-A\bar{x}}{EI}$$

$$= \frac{-\left[\frac{1}{2}(l)(R_B \times l)\right] \times \left[\frac{2}{3}(l)\right]}{EI}$$

$$\delta B_2 = \frac{-R_B l^3}{3EI}$$

Now $\delta B_1 + \delta B_2 = 0$

If there is no sinking of support.

$$\delta B_1 + \delta B_2 = 0$$

$$\Rightarrow \frac{5wl^3}{48EI} - \frac{R_B l^3}{3EI} = 0$$

$$\frac{5wl^3}{48EI} = \frac{R_B l^3}{3EI}$$

$$R_B = \frac{5w}{16}$$

$\Sigma V = 0 \Rightarrow R_A + R_B = W$

$$R_A + \frac{5w}{16} = w$$

$$R_A = w - \frac{5w}{16} = \frac{16w - 5w}{16}$$

$$R_A = \frac{11w}{16}$$

$$M_A = R_B \cdot l - w \times l/2 = \frac{5w}{16} \times l - w \times \frac{l}{2}$$

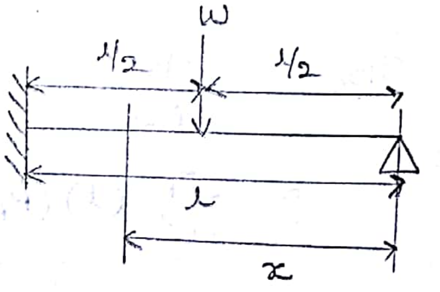
$$= \frac{5wl - 8wl}{16}$$

$$= \frac{-3wl}{16}$$

$$M_A = \frac{-3wl}{16}$$

M_A value is always negative.

Point of contraflexure:



To find the point of contraflexure. equate bending moment equation in the portion AC to '0'.

$$M_x = R_B \times x - w \times (x - l/2)$$

$$\Rightarrow 0 = \frac{5w}{16}x - wx + \frac{wl}{2}$$

$$\Rightarrow \frac{5wx}{16} - wx = \frac{-wl}{2}$$

$$\frac{5wx - 16wx}{16} = \frac{-wl}{2}$$

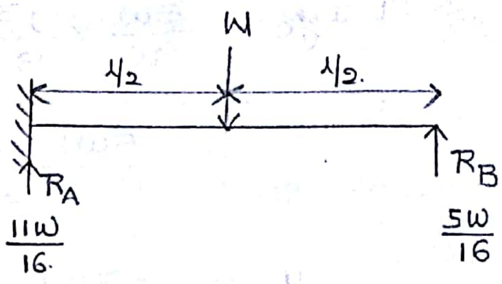
$$\Rightarrow \frac{\cancel{-}11wx}{16} = \frac{\cancel{-}wl}{2} \Rightarrow x = \frac{8l}{11}$$

In propped cantilever, point of contraflexture developed.

from fixed end $\frac{3l}{11}$

from prop. $\frac{8l}{11}$

Deflection equation:



Take the section in the portion AC at a distance x from "B" as shown in fig.

$$M_x = \frac{5W}{16}x - W(x - l/2).$$

$$\therefore EI\frac{d^2y}{dx^2} = M_x.$$

$$EI\frac{d^2y}{dx^2} = \frac{5W}{16}x - W(x - l/2)$$

$$EI\frac{dy}{dx} = \frac{5Wx^2}{16 \times 2} - \frac{W(x - l/2)^2}{2} + C_1$$

At $x = l$, $\frac{dy}{dx} = 0$

$$0 = \frac{5W}{32} \times l^2 - \frac{W}{2} \times \frac{l^2}{4} + C_1$$

$$C_1 = \frac{Wl^2}{8} - \frac{5Wl^2}{32} = \frac{4Wl^2 - 5Wl^2}{32}$$

$$\boxed{C_1 = \frac{-Wl^2}{32}}$$

$$EI.\frac{dy}{dx} = \frac{5Wx^2}{16 \times 2} - \frac{W(l - l/2)^2}{2} - \frac{Wl^2}{32} \longrightarrow (1)$$

Above equation.

$$EIy = \frac{5W}{32} \times \frac{x^3}{3} - \frac{W}{2}\frac{(x - l/2)^3}{3} - \frac{Wl^2}{32}x + C_2$$

At $x = 0$, $y = 0$

$$0 = 0 - 0 - 0 + C_2 \Rightarrow \boxed{C_2 = 0}$$

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$$EIy = \frac{5w}{32} \times \frac{x^3}{3} - \frac{w}{2} \frac{(x - l/2)^3}{3} - \frac{wl^2 x}{32} \longrightarrow ②$$

At $x = l/2$, $y = y_c$

$$EIy_c = \frac{5w}{32} \times \frac{(l/2)^3}{3} - \frac{w}{2} \frac{(l/2 - l/2)^3}{3} - \frac{wl^2(l/2)}{32}.$$

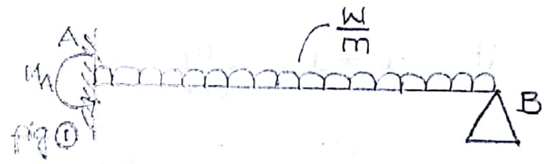
$$EIy_c = \frac{5w}{96} \times \frac{l^3}{8} - \frac{w}{6} \times 0 - \frac{wl^2}{32} \times \frac{l}{2}.$$

$$= \frac{5wl^3 - 12wl^3}{\cancel{486}.\ 768}$$

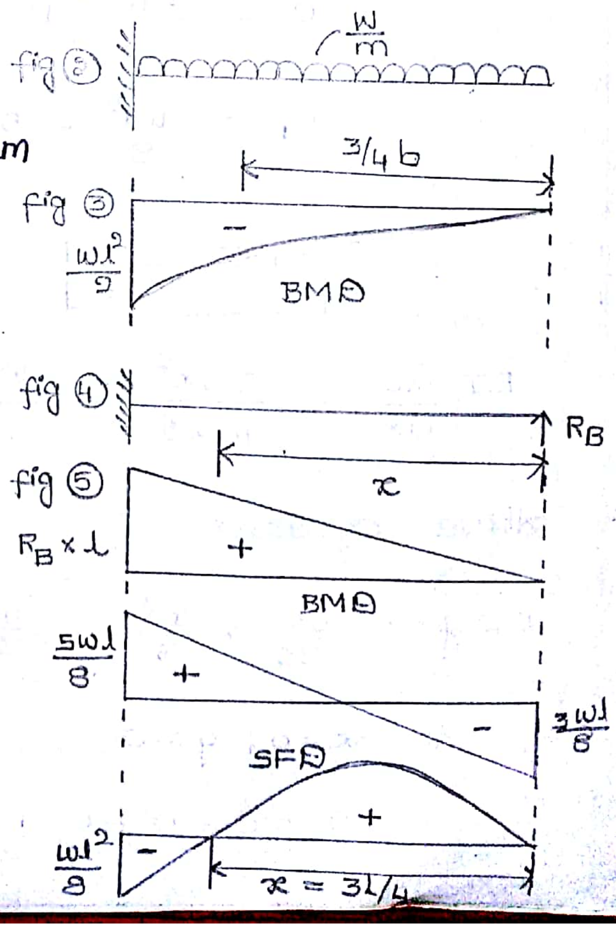
$$y_c = \frac{-7wl^3}{768\, EI}$$

* Analysing the propped cantilever beam having a uniformly distributed load (w) per unit length. The prop is to be given at the end of the beam. Find also the maximum deflection developed in the beam.

Analysing propped cantilever beam as shown in fig.



Remove the prop applied on the beam and draw the B.M diagram for the cantilever beam. (fig 2) as shown in fig ③



Find the deflection at the prop by moment area method.

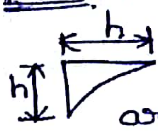
Let it be denoted as δB_1

δB_1 due to loading.

$$\delta B_1 = \frac{A\bar{x}}{EI}$$

$$= \frac{\frac{1}{3} \times l \times \frac{wl^2}{2} \times \frac{3}{4} l}{EI} = \frac{\frac{wl^3}{6} \times \frac{3}{4}}{EL}$$

Note



area $= \frac{1}{3} bh$.

$C.G = \frac{3}{4} b$

$$\delta B_1 = \frac{wl^4}{8EI}$$

Now remove the loading applied on the beam & Draw the bending moment diagram for the prop reaction let (R_B) as shown in fig ⑤. and find the deflection at the prop. Due to prop reaction (R_B) Let it be denoted as (δB_2).

$$\delta B_2 = \frac{-A\bar{x}}{EI}$$

$$= -\frac{\left(\frac{1}{2} \times l \times (R_B \times l)\right)\left(\frac{2}{3} l\right)}{EI}$$

$$\delta B_2 = \frac{-R_B l^3}{3EI}$$

Now $\delta B_1 + \delta B_2 = 0$

If there is no sinking at support.

$$\delta B_1 + \delta B_2 = 0$$

$$\frac{wl^4}{8EI} - \frac{R_B l^3}{3EI} = 0$$

$$\Rightarrow \frac{wl^4}{8EI} = \frac{R_B l^3}{3EI}$$

$$\frac{wl}{8} = R_B \times \frac{1}{3}$$

$$R_B = \frac{3wl}{8}$$

$\Sigma V = 0$

$$R_A + R_B = wl.$$

$$R_A + \frac{3wl}{8} = wl.$$

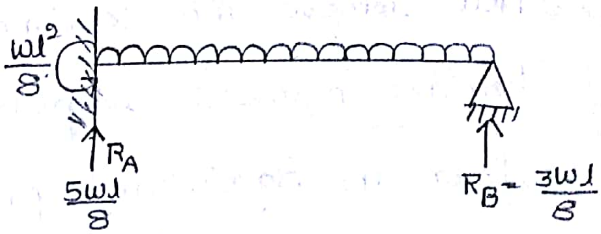
$$R_A = wl - \frac{3wl}{8} = \frac{8wl - 3wl}{8} = \frac{5wl}{8}$$

$$M_A = R_B \cdot L - wl \times \frac{l}{2}$$

$$= \frac{3wl}{8} \times l - \frac{wl^2}{2} = \frac{3wl^2}{8} - \frac{wl^2}{2} = \frac{3wl^2 - 4wl^2}{8}$$

$$M_A = \frac{-wl^2}{8}$$

M_A value always negative.



To find the point of zero.

shear force equate shear force equation in the portion BA to zero.

$$F_x = \frac{3wl}{8} - wx.$$

$$\Rightarrow 0 = \frac{3wl}{8} - wx \Rightarrow wx = \frac{3wl}{8}$$

$$x = \frac{3l}{8}$$

$$M_x = R_B \cdot x - wx \cdot \frac{x}{2}$$

$$= \frac{3wl}{8} \times \frac{3l}{8} - \frac{w}{2} \cdot \frac{9l^2}{64}.$$

$$= \frac{18wl^2 - 9wl^2}{128}$$

$$M_x = \frac{9wl^2}{128}$$

Point of contraflexure:

To find the point of contraflexure. equation. bending. equation. in the portion to zero.

$$M_x = R_B \cdot x - wx \cdot \frac{x}{2}$$

$$0 = \frac{3wl}{8} \cdot x - \frac{wx^2}{2}.$$

$$\frac{wx^2}{2} = \frac{3wl}{8} x$$

$$\Rightarrow x = \frac{3l}{4} \quad \text{from prop.}$$

Deflection equation:

$$M_x = R_B \cdot x - wx \cdot \frac{x}{2} = \frac{3wl}{8}x - \frac{wx^2}{2}$$

$$EI \cdot \frac{d^2y}{dx^2} = M_x .$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{3wlx}{8} - \frac{wx^2}{2}$$

Integrate the above equation.

$$E.I. \frac{dy}{dx} = \frac{3wl}{8} \cdot \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3} + c_1$$

Once again Integrate the above equation.

$$E.I. y = \frac{3wl}{8} \cdot \frac{x^3}{6} - \frac{w}{2} \cdot \frac{x^4}{12} + c_1 x + c_2$$

At $x = 0,\ y = 0$

$$0 = 0 - 0 + 0 + c_2$$

$$\Rightarrow c_2 = 0$$

At $x = l,\ \frac{dy}{dx} = 0$

$$EI(0) = \frac{3wl}{16}(l^2) - \frac{w}{6}(l^3) + c_1$$

$$= \frac{3wl^3}{16} - \frac{wl^3}{6} + c_1$$

$$c_1 = \frac{wl^3}{6} - \frac{3wl^3}{16}.$$

$$c_1 = \frac{8wl^3 - 9wl^3}{48}$$

$$c_1 = \frac{-wl^3}{48}$$

Deflection at the centre:

At $x = l/2,\ y = y_c.$

$$EIy = \frac{3wl}{8} \times \frac{x^3}{6} - \frac{w}{2} \times \frac{x^4}{12} - \frac{wl^3}{48}x .$$

$$EIy_c = \frac{3wl}{48}\left(\frac{l}{2}\right)^3 - \frac{w}{24}\left(\frac{l}{2}\right)^4 - \frac{wl^3}{48}\left(\frac{l}{2}\right)$$

$$= \frac{wl^4}{128} - \frac{wl^4}{384} - \frac{wl^4}{96}$$

$$= wl^4 \left(\frac{3-1-4}{384}\right)$$

$$= wl^4 \left(\frac{-2}{384}\right)$$

$$y_c = \frac{-wl^4}{192\,EI.}$$

* Analyse the propped cantilever beam as shown in fig & draw the SFD and BMD.

Remove the prop applied on the beam and draw the bending moment diagram through cantilever beam fig ② as shown in fig ③.

Find the deflection at the prop by moment area method.

Let it be denoted by δB_1

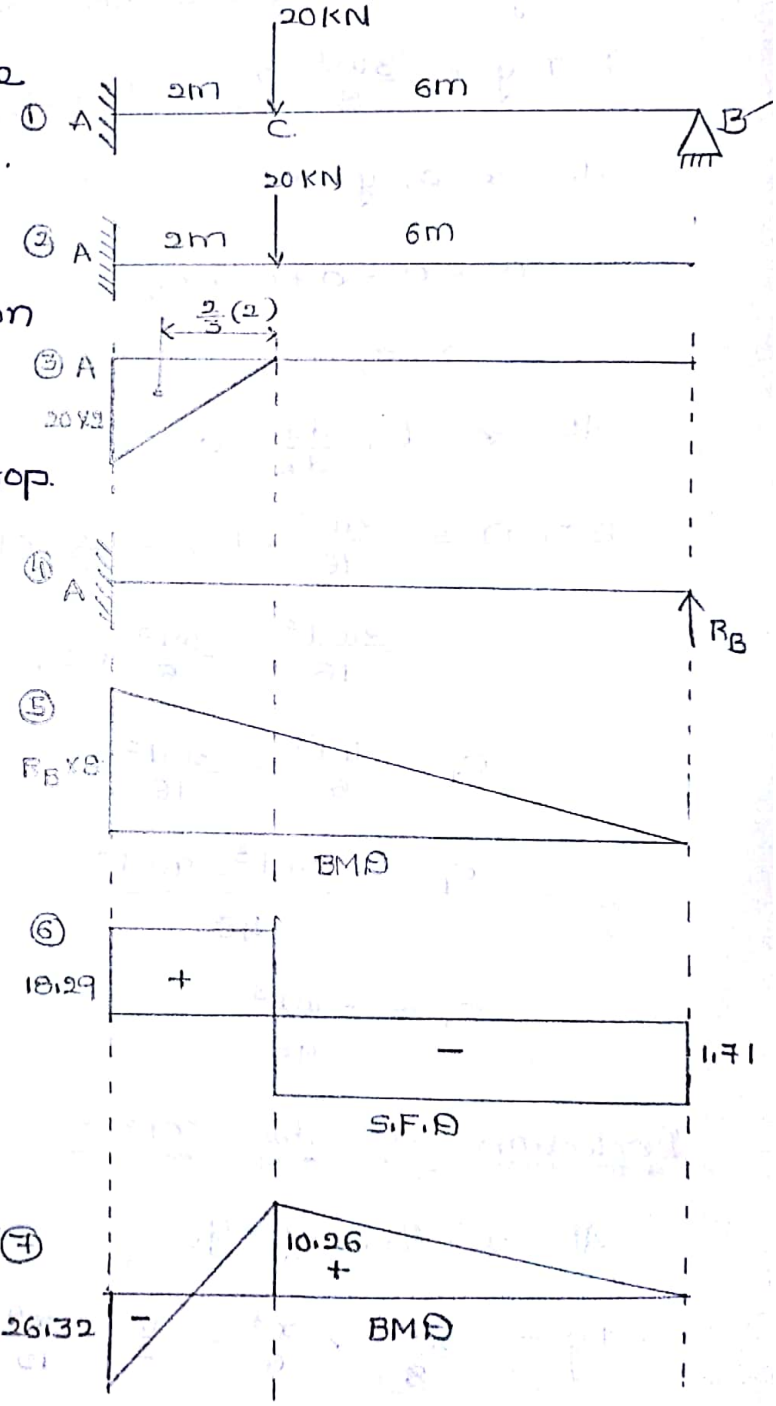
$$\delta B_1 = \frac{A\bar{x}}{EI.}$$

$$\delta B_1 = \frac{\left(\frac{1}{2} \times 40 \times 2\right)\left(\frac{2}{3} \times 2 + 6\right)}{EI.}$$

$$= \frac{40}{EI}\left(\frac{4}{3} + 6\right)$$

$$= \frac{40}{EI}\left(\frac{22}{3}\right)$$

$$\delta B_1 = \frac{880}{3EI.}$$



Now remove the loading applied on the beam & draw the bending moment diagram for the prop reaction 'R_B' as shown in fig 5 & find the deflection at the prop due to prop. reaction R_B. Let it be denoted as δB_2

$$\delta B_2 = \frac{-A\bar{x}}{EI}$$

$$= \frac{-\left(\frac{1}{2} \times 8R_B \times 8\right)\left(\frac{2}{3} \times 8\right)}{EI}$$

$$= \frac{-R_B \times 8^3}{3EI}$$

$$\delta B_2 = \frac{-512\, R_B}{3EI}$$

$\cancel{\delta B_2 =}$

$$\delta B_1 + \delta B_2 = 0$$

$$\frac{880}{3EI} - \frac{512\, R_B}{3EI} = 0$$

$$R_B = \frac{880}{512}$$

$$R_B = 1.71 \text{ KN.}$$

$\Sigma V = 0 \Rightarrow R_A + R_B = 20$ KN

$R_A + 1.71 = 20$

$R_A = 18.29$ KN

$\Sigma M_A = 0 \Rightarrow R_B \times 8 - (20 \times 2) = 0.$

$M_A = (1.71 \times 8) - (20 \times 2)$

$M_A = -26.32$ KN

$M_C = R_B \times 6$

$= 1.71 \times 6$

$= 10.26$ KN.

$M_C = 10.26$ KN.

Point of contraflexure:

To find the point of contraflexure equate BM equation in the portion CA to zero.

$$M_x = 1.71x - 20(x-6)$$

$$0 = 1.71x - 20x + 120$$

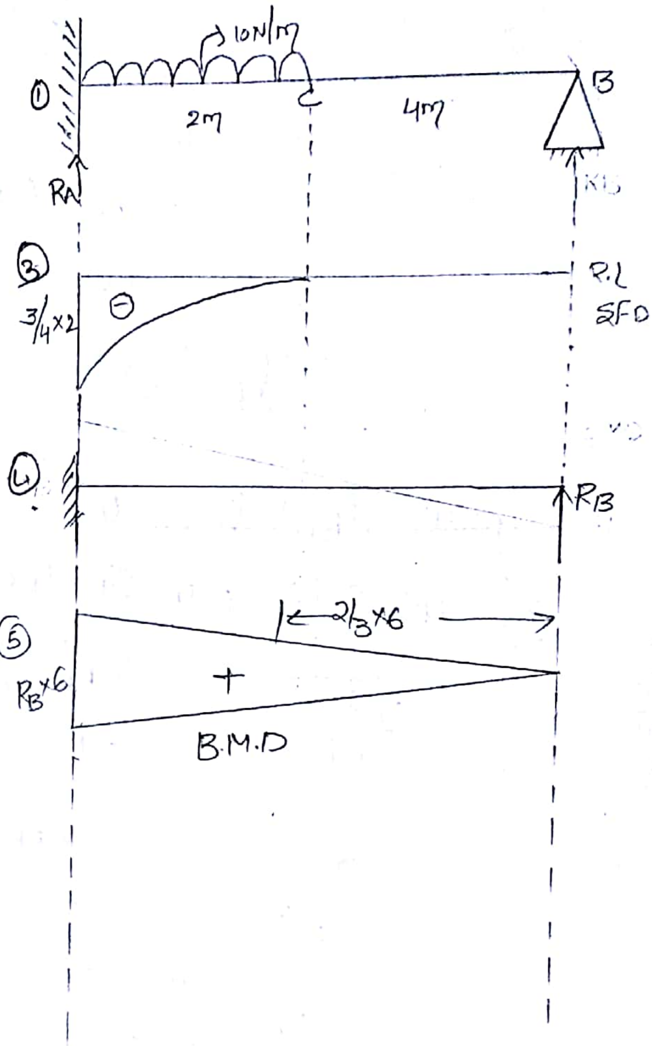
$$20x - 1.71x = 120$$

$$18.29x = 120$$

$$x = 6.56 \text{ m.}$$

① Analysed the propped cantilever beam as shown in figure,

Remove the prop uplyed on the beam & draw the B.m diagram for the cantilever beam (fig 3) as shown in fig (3)



find the deflection at the prop by moment area method let it B denoted as δ_{B_1} due to loading

$$\delta_{B_1} = \frac{A\bar{x}}{EI}$$

$$= \frac{\left(\frac{1}{3}\times 2\times 20\right)\left(\frac{3}{4}\times 2+4\right)}{EI}$$

$$= \frac{\left(\frac{40}{3}\right)\left(\frac{6+16}{4}\right)}{EI}$$

$$= \frac{\left(\frac{\cancel{40}^{10}}{3}\right)\left(\frac{22}{\cancel{4}}\right)}{EI} =$$

$$= \frac{73.33}{EI}$$

$$\delta_{B_1} = \frac{73.33}{EI}$$

Now remove the loading applyed on the Beam & draw the B.m diagram for the prop reaction let (RB) as shown in fig (5) and find the deflection at the prop.

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$$\delta_{B_2} = \frac{-A\bar{x}}{EI}$$

$$= -\frac{\left(\frac{1}{2} \times (R_B \times \cancel{6}^{3}) \times 6\right)\left(\frac{2}{3} \times 6\right)}{EI}$$

$$= -\frac{(\cancel{18}^{6} R_B)\left(\frac{12}{\cancel{3}}\right)}{EI}$$

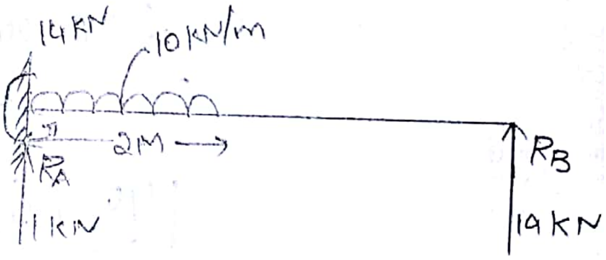
$$\delta_{B_2} = -\frac{72 R_B}{EI}$$

Now $\delta_{B_1} + \delta_{B_2} = 0$

If there is no sinking of support

$$\frac{73.33}{EI} - \frac{72 R_B}{EI} = 0$$

$$\frac{73.33}{\cancel{EI}} = \frac{72 R_B}{\cancel{EI}}$$



$$R_B = \frac{73.33}{72}$$

$$\boxed{R_B = 1 \text{ kN}}$$

$\Sigma_V = 0$ $\quad R_A + R_B = 20$

$$R_A + 1 = 20$$

$$R_A = 20 - 1$$

$$\boxed{R_A = 19 \text{ kN}}$$

$$M_A = R_B \times 6 - 10 \times 2 \times \frac{2}{2}$$

$$= 6 - 20$$

$$\boxed{M_A = -14 \text{ kN-m}}$$

(8)

$$M_C = R_B \times 4$$
$$= 1 \times 4$$
$$= 4\ kN\text{-}m$$

portion BC (0 to 4m)

$$F_x = -R_B$$
$$= -1 kN$$

$$M_x = R_B \times x$$

$$M_{x=0} = 0$$

$$M_{x=4} = 4\ kN\text{-}m$$

portion CA (4 to 6m)

$$F_x = -R_B + 10(x-4)$$

$$F_{x=4} = -1$$

$$F_{x=6} = -1+20$$
$$= 19\ kN$$

$$M_x = R_B \times x - 10(x-4)\left(\frac{x-4}{2}\right)$$

$$M_{x=4} = 1 \times 4 = 4$$

$$M_{x=6} = 1 \times 6 - 10(2)\left(\frac{2}{2}\right)$$
$$= -14\ kN\text{-}M$$

To find the point of zero shear force equate fx equation in the portion CA to zero

$$F_x = -R_B + 10(x-4)$$
$$0 = -1 + 10(x-4)$$
$$10x - 40 - 1 = 0$$

$$x = \frac{41}{10}$$

$$\boxed{x = 4.1}$$

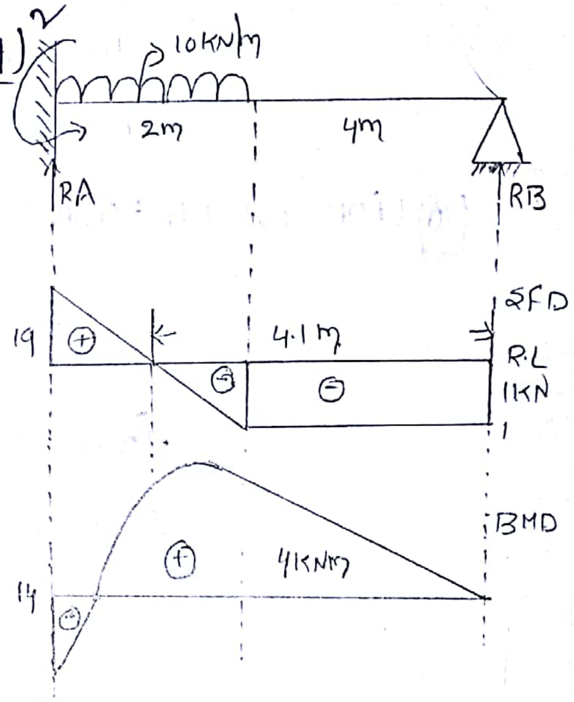
To find the maximum B.m apply $x = 4.1\,m$ In the B.m equation portion CA

$$m_x = R_B \times x - 10\,(x-4)\,\frac{(x-4)}{2}$$

$$0 = 1 \times 4.1 - \cancel{10}^{5}\,\frac{(4.1-4)^2}{\cancel{2}}$$

$$= 4.1 - 0.05$$

$$m_x = 4.05$$



deflection equation:-

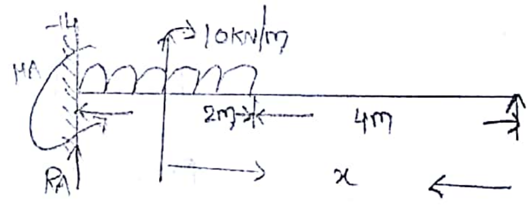
$$m_x = x - 10\,(x-4)\,\frac{(x-4)}{2}$$

$$EI\frac{d^2y}{dx^2} = m_x$$

$$EI\frac{d^2y}{dx^2} = x - 10\,(x-4)\,\frac{(x-4)}{2}$$

$$EI\frac{dy}{dx} = \frac{x^2}{2} - 5\,\frac{(x-4)^3}{3} + C_1$$

$$EIy = \frac{x^3}{6} - 5\,\frac{(x-4)^4}{12} + C_1x + C_2$$



At $x = 0$, $y = 0$

$$0 = 0 - 0 + 0 + C_2$$

$$\boxed{C_2 = 0}$$

At $x = 6m$ $\frac{dy}{dx} = 0$

$$0 = \frac{6^2}{2} - 5\,\frac{8}{3} + C_1$$

9

$$\boxed{c_1 = -4.66}$$

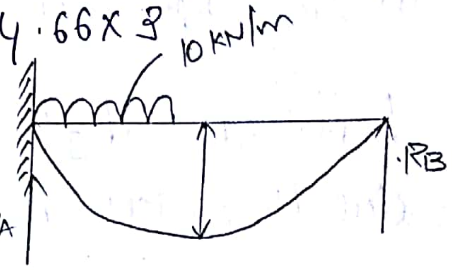
$$EI\,y = \frac{x^3}{6} - 5\,\frac{(x-4)^4}{12} - 4.66x \rightarrow (1)$$

Deflection at the center

At $x = 3m$, $y = y_c$

$$EI\,y_c = \frac{3^3}{6} - 5\,\frac{(3-4)^4}{12} - 4.66 \times 3$$

$$\boxed{y_c = \frac{-9.48}{EI}}$$



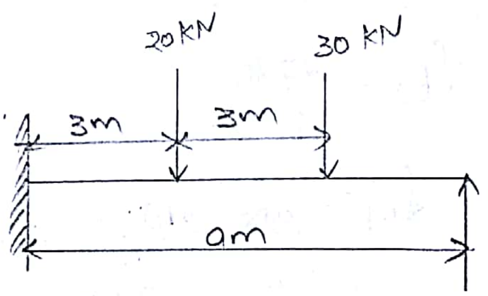
2 A cantilever Beam of length 9m carries 2 point loads of 20KN, 30KN at 1/3 points from fixed supports the canti-lever is supported at the free end calculate?

(i) prop reaction

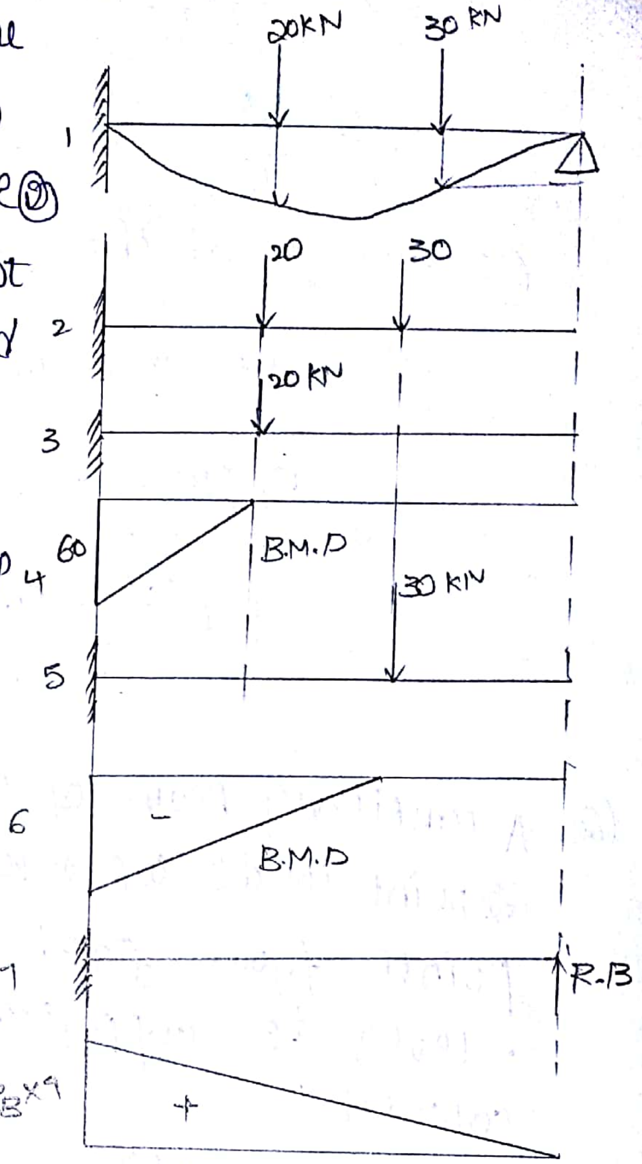
(ii) moment at fixed support

(iii) deflection under the point loads

Take $E = 2 \times 10^5\ N/mm^2$ & $I = 85 \times 10^5\ N/m^4$



Sol: Remove the prop applied on the Beam & Draw the B.M diagram for the cantilever Beam. find the deflection at the prop by moment area method let it be denoted as δB_1, $\delta B_1 = \delta b_1 + \delta b_2$



hence δb_1 = deflection at the prop due to 20 KN Load

$$\delta b_1 = \frac{A\bar{x}}{EI}$$

$$= \frac{\left(\frac{1}{2}\times 3\times \cancel{60}^{30}\right)\left(\frac{2}{3}\times 3+6\right)}{EI}$$

$$= \frac{90\times 8}{EI} = \frac{720}{EI}$$

δb_2 = deflection at the prop due to 30 KN load $\delta b_2 = \frac{A\bar{x}}{EI} =$

$$\delta b_2 = \frac{A\bar{x}}{EI} = \frac{\left(\frac{1}{2}\times 6\times \cancel{180}^{90}\right)\left(\frac{2}{3}\times \cancel{6}^{2}+3\right)}{EI}$$

$$= \frac{540\times 7}{EI}$$

$$\delta b_2 = \frac{3780}{EI}$$

$$\delta B_1 = \delta b_1 + \delta b_2$$

$$= \frac{720}{EI} + \frac{3780}{EI}$$

$$\boxed{\delta B_1 = \frac{4500}{EI}}$$

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(10) Now remove the load applied on the beam draw the bending moment diagram for the prop

$$\delta B_2 = \delta b_1 + \delta b_2$$

here δb_1 = deflection at the prop due to 20 kN Load

$$\delta B_2 = -\frac{A\bar{x}}{EI}$$

$$= -\frac{\left(\frac{1}{2} \times \overset{4.5}{\cancel{9}} \times R_B 9\right)\left(\frac{L}{3} \times \cancel{9}^{3}\right)}{EI}$$

$$\boxed{\delta B_2 = \frac{-243 R_B}{EI}}$$

$$\delta B_1 + \delta B_2 = 0$$

$$\frac{4500}{EI} - \frac{243 RB}{EI} = 0$$

$$4500 - 243 RB$$

$$RB = \frac{4500}{243}$$

$$\boxed{R_B = 18.51}$$

$$R_A + R_B = 50 \text{ kN}$$

$$RA + 18.51 = 50$$

$$RA = 50 - 18.51$$

$$\boxed{RA = 31.49 \text{ kN}}$$

(i) prop reaction $R_B = 18.51$

moment at A :-

$$\Sigma m A = 0$$

$$R_B \times 9 - 30 \times 6 - 20 \times 3 = 0$$

$$18.51 \times 9 - 180 - 60 = 0$$

$$\boxed{M_A = -73.41}$$

(ii) moment at fixed support $M_A = -73.41$

(iii) deflection under the load

$$M_x = R_B x - 30(x-3) - 20(x-6)$$

i. $$EI \frac{d^2y}{dx^2} = M_x$$

$$EI \frac{d^2y}{dx^2} = 18.51x - 30(x-3) - 20(x-6)$$

$$EI \frac{dy}{dx} = 18.51 \frac{x^2}{2} - 30 \frac{(x-3)^2}{2} - 20 \frac{(x-6)^2}{2} + C$$

$$EIy = 18.51 \frac{x^3}{6} - 30 \frac{(x-3)^3}{6} - 20 \frac{(x-6)^3}{6} + C_1 x + C_2$$

At $x = 0$, $y = 0$

$$0 = 0 - 0 - 0 + C_2$$

$$\boxed{C_2 = 0}$$

At $x = 9$ $\frac{dy}{dx} = 0$

$$0 = 18.51 \frac{(9)^2}{2} - 30 \frac{(9-3)^2}{2} - 20 \frac{(9-6)^2}{2} + C_1$$

$$= 18.51 \left(\frac{81}{2}\right) - 30 \frac{(6)^2}{2} - 30 \frac{(3)^2}{2} + C_1$$

$$\boxed{C_1 = -119.6}$$

(11)

$$EIy = 18.51\frac{x^3}{6} - 30\frac{(x-3)^3}{6} - 20\frac{(x-6)^3}{6} - 119.6x \rightarrow ①$$

at $x = 3m$ $y = y_C$

$$EIy_C = 18.51\frac{(3)^3}{6} - 0 - 0 - 119.6\times 3$$

$$y_C = \frac{-275.5}{EI}$$

$$= \frac{-275.5}{1700}$$

$$= -0.16m$$

$$= 160mm$$

$$EI = 2\times 10^5 \times 85\times 10^5 \ N\text{-}mm^2$$

$$= 2\times 85\times 10^{10}\times \frac{1}{10^3}\times\frac{1}{(10^3)^2} \ kN\text{-}m^2$$

$$= 1700$$

at $x = 6m$ $y = y_D$

$$EIy_D = 18.51\frac{(6)^3}{6} - 30\frac{(6-3)^3}{6} - 0 - 119.6\times 6$$

$$EIy_D = 666.36 - 135 - 717.6$$

$$y_D = \frac{-186.24}{EI} = \frac{-186.24}{1700}$$

$$y_D = -0.109$$

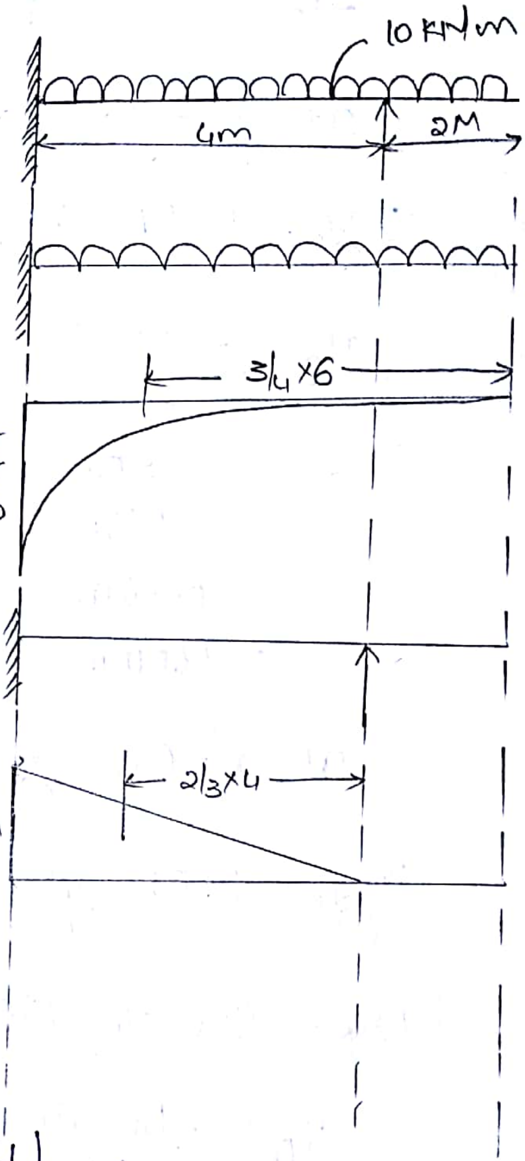
$$y_D = 109\,mm$$

(3) A canticey beam of length 6m carries a U.D.L of 10 kN/m through out the Beam the cantilever is supported at 2m from free end calculate

i) prop reaction

ii) moment at A

Sol: Remove the prop applied on the beam & draw the B.M.D for cantilever beam. fig(3) find the deflection at the prop by moment area method let it be denoted as δ_{B_1}



$$\delta_{B_1} = \frac{A\bar{x}}{EI}$$

$$= \frac{(\frac{1}{3} \times 6 \times 180)(\frac{3}{4} \times 6 - 2)}{EI}$$

$$= \frac{900}{EI}$$

$$\boxed{\delta_{B_1} = \frac{900}{EI}}$$

$$\delta_{B_2} = -\frac{A\bar{x}}{EI}$$

$$= -\frac{(\frac{1}{2} \times 4 \times R_B \times 4)(\frac{2}{3} \times 4)}{EI}$$

$$= \frac{-8R_B \times \frac{8}{3}}{EI}$$

$$= -\frac{21.33 R_B}{EI}$$

$$\boxed{\delta_{B_2} = \frac{-21.33 R_B}{EI}}$$

$$\delta_{B_1} + \delta_{B_2} = 0$$

$$\frac{900}{EI} - \frac{21.33 R_B}{EI} = 0$$

$$\frac{900}{\cancel{EI}} = \frac{21.33 R_B}{\cancel{EI}}$$

$$EI\frac{dy}{dx} = R_B\frac{(x-2)^2}{2} - \frac{10x^3}{6} + C_1$$

$$0 = R_B\frac{(6-2)^2}{2} - 10\frac{(6)^3}{6} + C_1$$

$$= 42.19 \times \frac{4^2}{2} - 10\times 6^2 + C_1$$

$$-C_1 = -22.48$$

$$\boxed{C_1 = 22.48}$$

C_1 value substitute in eq (i)

$$2C_1 + C_2 - 6.667 = 0$$

$$2(22.48) + C_2 - 6.667 = 0$$

$$C_2 = 6.667 - 44.96$$

$$\boxed{C_2 = -38.29}$$

$$EI\frac{dy}{dx} = R_B\left(\frac{x-2}{2}\right)^2 - 10\frac{(x)^3}{6} + 22.48$$

$$EIy = R_B\frac{(x-2)^3}{6} - \frac{10x^4}{24} + 22.48 - 38.29$$

Q) a cantilever of length 8m carries a central point load of 10 kN during a central point load of 10 kN during loading the support at B (free) sinker by 10 mm loading
calculate

(i) prop reaction

(ii) moment at A

$E = 2\times10^5$ N/mm^2

$I = 85\times10^5$ mm^4

$$\frac{900}{21.33} = R_B$$

$$\boxed{R_B = 42.19\,KN}$$

$\Sigma V = 0 \quad R_A + R_B = 10 \times 6$

$R_A = 60 - 42.19$

$$\boxed{R_A = 17.81\,KN}$$

i) prop reaction $= 42.19$ KN

ii) <u>moment at fixed end</u>:-

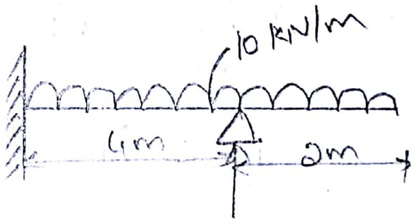
$M_x = R_B \times 4 - 10 \times 6 \times 6/2$

$= 42.19 \times 4 - 180$

$= -11.24$

$$\boxed{M_x = -11.24\,KN\text{-}m}$$

(iii) <u>Deflection</u>:-



$M_x = R_B(x-2) - 10(x)\left(\frac{x}{2}\right)$

$EI\frac{d^2y}{dx^2} = R_B(x-2) - 10\frac{x^2}{2}$

$EI\frac{dy}{dx} = R_B\frac{(x-2)^2}{2} - \frac{10x^3}{6} + C_1$

$EIy = R_B\frac{(x-2)^3}{6} - \frac{10x^4}{24} + C_1 x + C_2$

At $x = 2m$, $y = 0$

$0 = R_B\frac{(2-2)^3}{6} - 10\frac{(2)^4}{24} + C_1(2) + C_2$

$= 0 - 6.667 + 2C_1 + C_2$

$2C_1 + C_2 = 6.667 = 0 \longrightarrow (1)$

$x = 6m, \ \frac{dy}{dx} = 0$

The prop due to prop reaction R_B let it be denoted as

$$\delta B_2 = -\frac{A\bar{x}}{EI}$$

$$= -\frac{\left(\frac{1}{2} \times R_B \times 6 \times 6\right)\left(\frac{2}{3} \times 6\right)}{EI}$$

$$= -\frac{R_B \times 6^3}{3EI} = \frac{-72 R_B}{EI}$$

$$\delta B_1 + \delta B_2 = 0$$

$$\frac{864}{EI} - \frac{72 R_B}{EI} = 0$$

$$-72 R_B = -864$$

$$R_B = \frac{-864}{-72}$$

$$\boxed{R_B = 12 \text{ kN}}$$

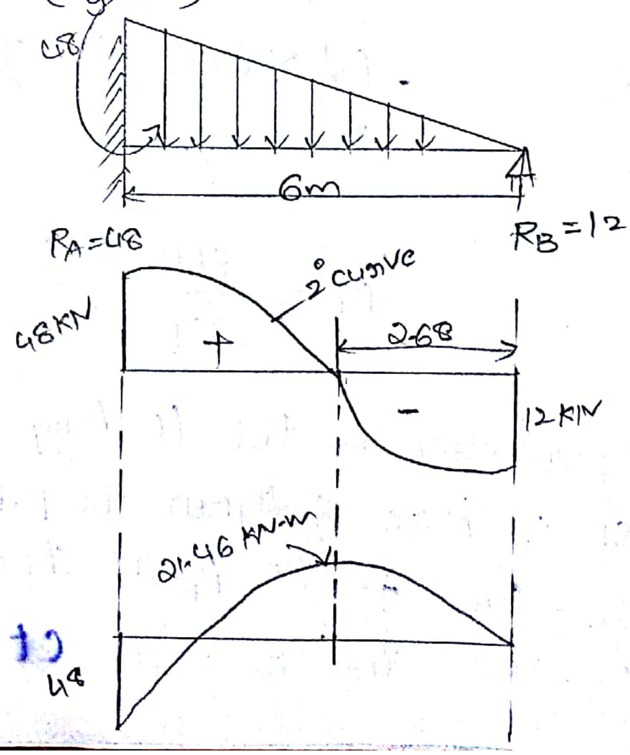
$$R_A + R_B = \frac{1}{\cancel{2}} \times \cancel{20}^{10} \times 6$$

$$12 + R_A = 60$$

$$\boxed{R_A = 48 \text{ kN}}$$

$$M_A = R_B \times 6 - \left(\frac{1}{2} \times 60 \times 6\right)\left(\frac{1}{3} \times 6\right)$$

$$M_A = -48 \text{ kN-m}$$



(4) (ii) moment at A :-

ΣM_A

$R_B \times 8 - 10 \times 4 = 0$

$3.125 \times 8 - 40 = 0$

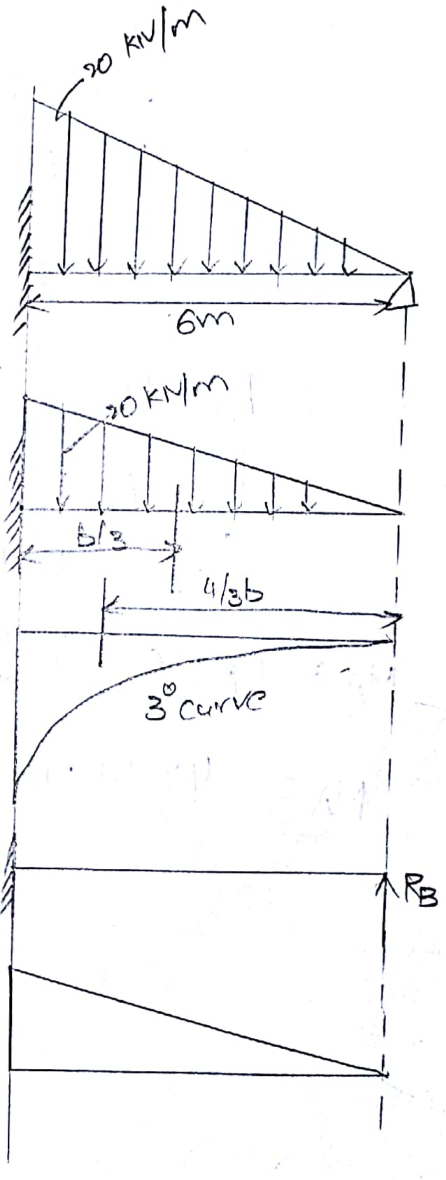
$\boxed{M_A = -15 \text{ kN-m}}$

$M_C = R_B \times 4$

$= 3.125 \times 4$

$\boxed{M_C = 12.5 \text{ kN-m}}$

(iii) moment at A = $M_A = -15$ kN-m



① Remove the prop applied on the beam and draw the BMD through cantilever beam (fig 2 as shown in fig (3) find the deflection at the prop by moment area method. let it be denoted as δ_{B_1}

$$\delta_{B_1} = \frac{A\bar{x}}{EI}$$

$$= \frac{\left(\frac{1}{4} \times 6 \times 60 \times \frac{6}{3}\right)\left(\frac{4}{5} \times 6\right)}{EI}$$

$$\delta_{B_1} = \frac{864}{EI}$$

Now remove the loading applied on the beam & draw the B.M.D for the prop reaction R_B as shown in fig (5) & find the deflection at

$$\delta_{B2} = -\frac{A\bar{x}}{EI}$$

$$= -\frac{\left(\frac{1}{2} \times \cancel{8}^{4} \times RB \times 8\right)\left(\frac{2}{3} \times 8\right)}{EI}$$

$$= -\frac{RB \times 8^3}{3EI}$$

$$= -\frac{512 RB}{3EI}$$

$$\delta_{B2} = \frac{-512 R_B}{3EI}$$

$$\delta_{B1} + \delta_{B2} = 0$$

$$\frac{533.33}{EI} + \left(\frac{-512 \times RB}{3EI}\right) = 0$$

$$\frac{533.33}{EI} - \frac{512 \times R_B}{3EI} = 0$$

$$\frac{1600}{3\cancel{EI}} - \frac{512 RB}{\cancel{3EI}} = 0$$

$$1600 = 512 R_B$$

$$\Rightarrow R_B = \frac{1600}{512}$$

$$\boxed{R_B = 3.125 \text{ M}}$$

$\Sigma V = 0$ $RA + R_B = 10$

$RA + 3.125 = 10$

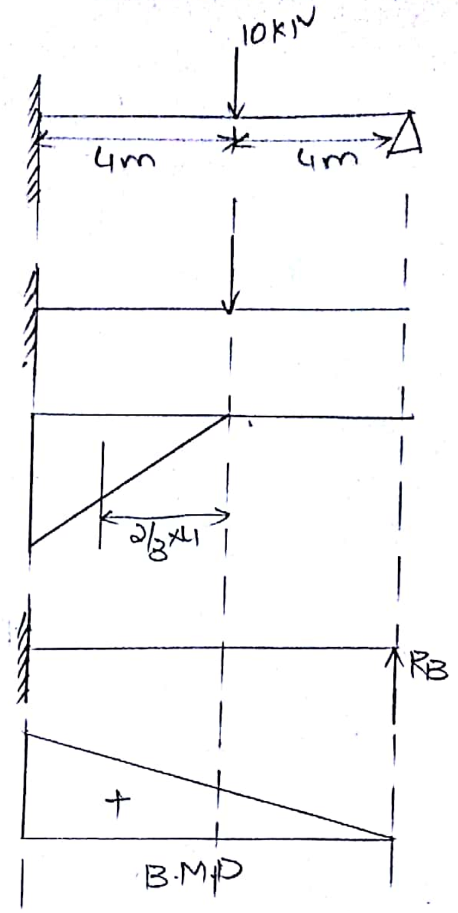
$RA = 10 - 3.125$

$RA = 6.875$ m

$\therefore$ $\boxed{\text{prop reaction } RB = 3.125 \text{ m}}$

(13)

Sol: Remove the prop applied on the beam and draw the bending moment diagram through cantilever beam (fig 3) find the deflection at the prop by moment area method let area method. let it be denoted as δB_1



$$\delta_{B1} = \frac{A\bar{x}}{EI}$$

$$= \frac{(\frac{1}{2} \times 40 \times 4)(\frac{2}{3} \times 4 + 4)}{EI}$$

$$= \frac{80 \times 6.66}{EI}$$

$$\delta_{B_1} = \frac{A\bar{x}}{EI} = (\frac{1}{2} \times 4 \times 40)(\frac{2}{3} \times 4 + 4)$$

$$= \frac{80(\frac{8}{3} + 4)}{EI}$$

$$\delta_{B1} = \frac{80}{EI}(\frac{20}{3})$$

$$\delta_{B_1} = \frac{533.33}{EI}$$

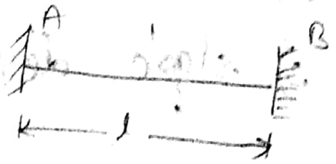
$$\Rightarrow \boxed{\delta_{B_1} = \frac{1600}{3EI}}$$

Now remove the loading applied on the beam and draw the bending moment diagram for the prop reaction "RB" as shown bending moment diagram for the prop reaction (5) & find the deflection at the prop due to prop reaction RB. let it be denoted as δB_2

UNIT- 2

(41) # Fixed beams (1)

Fixed beams :- when the beam fixed at both the ends then the beam is called as fixed beam.



The fixed beam having 6 unknowns reactions components, 3 at each supports. The degree of indetermancy considering the horizontal forces.

$$D_S = r - 3$$
$$= 6 - 3$$
$$= 3$$

The degree of static indetermancy not considering the horizontal forces.

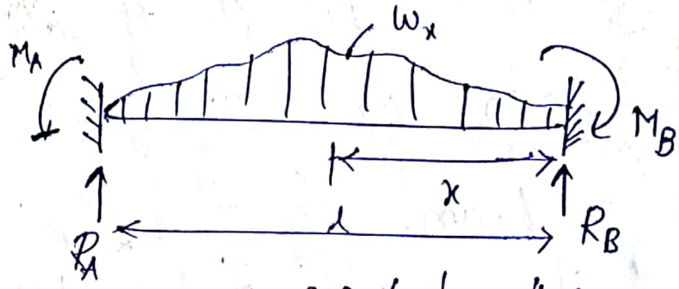
$$D_S = r - 2$$
$$= 4 - 2$$
$$= 2$$

Degree of kinematic indetermancy is 'zero' for fixed beams because their is no responses for slopes & deflections

6

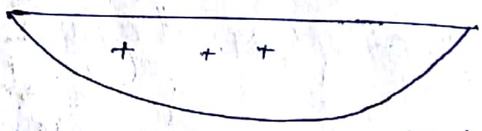
Fig (a) shows a fixed beam AB of uniform section and span l, loaded as shown in figure. As the ends of the beam are fixed, the slopes at the supports after loading will be zero as shown in deflected form

Let M_A and M_B be the fixing moments at the supports A and B respectively.



(a) load diagram


(b) Deflected form


(c) Simply supported B.M dia

(d) BM diagram due to fixed end moments.

The angle between the two tangents drawn at A and B on the deflected curve is zero therefore, the total area of M/EI diagram between A and B will be zero. For a beam of constant moment of inertia $A/EI = 0$

where A is the total area of B.M. diagram.

The fixed ~~end~~ Beam can be looked upon as simply supported beam with end moments M_A and M_B such that the slopes at the supports are zero.

Due to simply supported condition the loading will cause +ve B.M. Due to fixing moments the B.M. diagram will vary from M_A at A to M_B at B. As the total area of M/EI diag. is to be zero, the area of M/EI diag. due to fixing moments will be equal to the area of M/EI diag. as simply supported beam.

For a beam of constant moment of inertia, if A_s is the area of B.M. diagram considering beam as simply supported and A_i is the area of B.M. diagram due to fixing moments.

$$\therefore \frac{A}{EI} = \frac{A_s}{EI} + \frac{A_i}{EI} = 0$$

$$A_i = -A_s$$

Where $A_i = \frac{(M_A + M_B)}{2} \times l$

$$\therefore \frac{M_A + M_B}{2} \times l = -A_s$$

$$\boxed{M_A + M_B = -\frac{2A_s}{l}} \longrightarrow (i)$$

The tangent drawn at A will pass through B, therefore, the intercept on the vertical at A by the tangents drawn at A and B will be zero. Therefore moment of M/EI diag. about A will be zero. Similarly moment of M/EI dia. about B will be zero.

$$\therefore \frac{A\bar{x}}{EI} = 0$$

where $\bar{x}$ is the distance of the C.G. of B.M. diagram area from the support.

$$\frac{A_s}{EI}\bar{x}_s + \frac{A_i}{EI}\bar{x}_i = 0$$

$\bar{x}_s$ and $\bar{x}_i$ are the distances of centre of gravity of A_s and A_i respectively, from end A.

$$\therefore A_i\bar{x}_i = M_A \times \frac{l}{2} \times \frac{l}{3} + M_B \times \frac{l}{2} \times \frac{2l}{3}$$

$$= \frac{l^2}{6}(M_A + 2M_B) = -A_s\bar{x}_s$$

$$\boxed{M_A + 2M_B = -\frac{6A_s\bar{x}_s}{l^2}} \longrightarrow (2)$$

Solving (1) and (2)

$$M_B = -\frac{6A_s\bar{x}_s}{l^2} + \frac{2A_s}{l^2} = \frac{-3A_s}{l^2}(3\bar{x}_s - l)$$

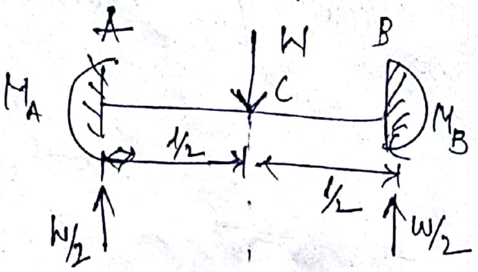
$$M_A = -\frac{2A_s}{l} + \frac{2A_s}{l^2}(3\bar{x}_s - l)$$

$$M_A = -\frac{2A_s}{l^2}(2l - 3\bar{x}_s)$$

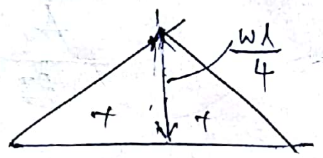
Fixing moment for a fixed beam of uniform section due to

i. concentrated load at the centre of span :

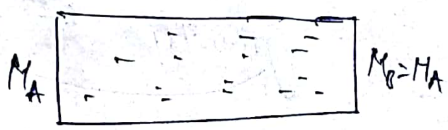
Consider a fixed ended beam AB with a concentrated load W acting at the mid span.



As a simply supported beam M/EI diagram will be a triangle with maximum ordinate $\frac{Wl}{4EI}$ at the centre. As the Beam is symmetrical fixing moment M_A and M_B will be equal.



Simply supported B.M dia.



B.M. due to fixed end moment

$$A_s + A_i = 0$$

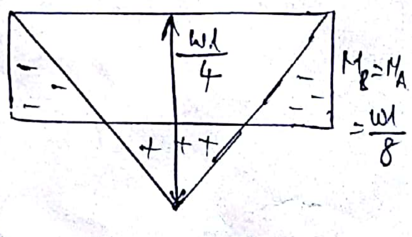
$$\left(\frac{1}{2} \times \frac{Wl}{4} \times l\right) + \left(\frac{M_A + M_B}{2}\right) l = 0$$

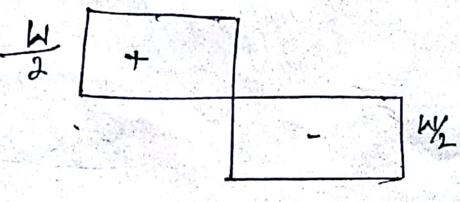
$$M_A + M_B = -\frac{Wl}{4}$$

But

$$M_A = M_B$$

$$M_A = M_B = -\frac{Wl}{8}$$





$$R_A = R_B = \frac{W}{2}$$

B.M. at a section x distance from A, between A and C will be

$$M_x = \frac{W}{2}x - \frac{Wl}{8} = \frac{W}{8}(4x - l)$$

At point of contraflexure

$$M_x = 0$$

$$\therefore \frac{w}{8}(4x - l) = 0$$

$$\boxed{x = \frac{l}{4}}$$

$\therefore$ The point of contraflexure will be at $l/4$ from either end.

Max. +ve B.M $= \frac{wl}{4} - \frac{wl}{8} = \frac{wl}{8}$

Max. −ve B.M. $= \frac{-wl}{8}$

Slope and deflection :-

At any section in AC distant x from the end A, the B.M. is given by, $M_x = \frac{w}{2}x - \frac{wl}{8}$

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}x - \frac{wl}{8}$$

Integrating, we get $EI\frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl}{8}x + C_1$

At $x = 0$, $\frac{dy}{dx} = 0$

$\therefore 0 = 0 - 0 + C_1 \Rightarrow C_1 = 0$

Integrating again $EIy = \frac{wx^3}{12} - \frac{wlx^2}{16} + C_2$

At $x = 0$, $y = 0$ $\boxed{C_2 = 0}$

∴ Deflection equation

$$EIy = \frac{wx^3}{12} - \frac{wlx^2}{16}$$

Max. deflection occurs at midspan

At $x = \frac{l}{2}$ $\quad y = y_c$

$$EIy_c = \frac{w}{12}\left(\frac{l}{2}\right)^3 - \frac{wl}{16}\left(\frac{l}{2}\right)^2$$

$$= \frac{wl^3}{96} - \frac{wl^3}{64} = -\frac{wl^3}{192}$$

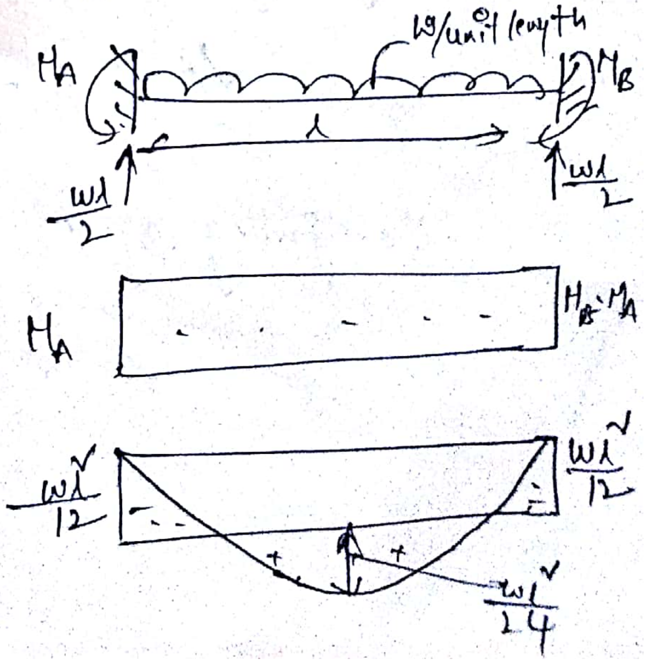
$$\boxed{y_c = -\frac{wl^3}{192EI} \text{ (Downward)}}$$

ii, uniformly distributed load of intensity 'w' per unit length throughout the span.

Consider a fixed ended beam AB with uniformly distributed load w/unit length througout the span.

As the loading is symmetrical about the centre, fixing moments M_A and M_B will be equal.

Simply supported B.M. diagram will be parabola with central ordinate $\frac{wl^2}{8}$



$$A_s + A_i = 0$$

$$\left(\frac{2}{3}\frac{wl^2}{8} \times l\right) + \left(\frac{M_A + M_B}{2}\right) l = 0$$

$$M_A + M_B = \frac{-wl^2}{6}$$

But $M_A = M_B$

$$\therefore \quad M_A = M_B = \frac{-wl^2}{12}$$

$$R_A = R_B = \frac{wl}{2}$$

B.M. at a section 'x' distance from A will be

$$M_x = \frac{wl}{2}x - \frac{wx^2}{2} - \frac{wl^2}{12}$$

At point of contraflexure B.M. is zero.

$$\frac{wl}{2}x - \frac{wx^2}{2} - \frac{wl^2}{12} = 0$$

$$x^2 - lx + \frac{l^2}{6} = 0$$

$$x = \frac{l \pm \sqrt{l^2 - 4 \cdot 1 \cdot \frac{l^2}{6}}}{2} = \frac{l \pm l\sqrt{\frac{1}{3}}}{2}$$

$$= \frac{l}{2}(1 \pm 0.577)$$

$$= 0.212l \text{ or } 0.788l$$

$\therefore$ points of contraflexure are at a distance of 0.212l from either end.

Max. +ve B.M. will be at the centre of the span.

$$\text{Max. +ve B.M} = \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24}$$

Max. -ve B.M. will be at the supports $= \frac{-wl^2}{12}$

Deflection and slope equations

B.M. at any section $= \frac{wl}{2}x - \frac{wl^2}{12} - \frac{wx^2}{2}$

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = \frac{wl}{2}x - \frac{wx^2}{2} - \frac{wl^2}{12}$$

Integrating, $EI\frac{dy}{dx} = \frac{wl}{4}x^2 - \frac{w}{2}\frac{x^3}{3} - \frac{wl^2}{12}x + C_1$

At $x=0$, $\frac{dy}{dx}=0$, $\boxed{0 = C_1}$

Integrating again, $EIy = \frac{wl}{4}\frac{x^3}{3} - \frac{w}{6}\cdot\frac{x^4}{4} - \frac{wl^2}{12}\frac{x^2}{2} + C_2$

At $x=0$, $y=0$, $0 = 0 + C_2$ $\boxed{C_2 = 0}$

$$\therefore \quad EIy = \frac{wl}{12}x^3 - \frac{w}{24}x^4 - \frac{wl^2x^2}{24}$$

For the deflection at the centre, putting $x = l/2$ in the deflection equation, we get

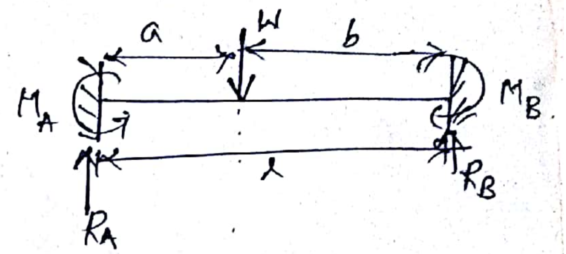
$$EIy_c = \frac{wl}{12}\left(\frac{l}{2}\right)^3 - \frac{w}{24}\left(\frac{l}{2}\right)^4 - \frac{wl^2}{24}\left(\frac{l}{2}\right)^2$$

$$= \frac{-wl^4}{384}$$

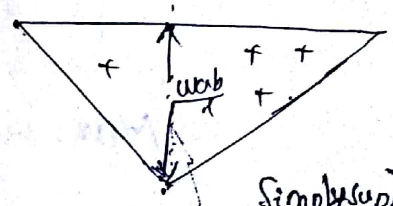
$$\boxed{y_c = -\frac{wl^4}{384EI}}$$

iii Unsymmetrical concentrated load

Consider a fixed ended beam AB with a concentrated load W acting at a distance "a" from the support A.



B.M. diagram as a simply supported beam will be a triangle with maximum ordinate under the load equal to $\frac{Wab}{l}$

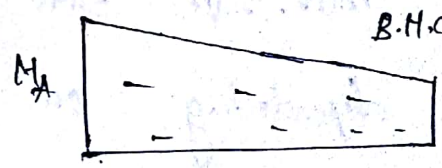


Simply supported B.M. diag.

$$M_A + M_B = -\frac{2A_s}{l}$$

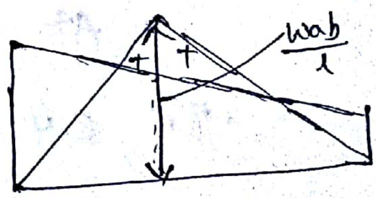
$$M_A + M_B = \frac{-2}{l}\left(\frac{1}{2} \times \frac{wab}{l} \times l\right)$$

$$= \frac{-wab}{l} \longrightarrow (1)$$



B.M. diagr. due to fixed end moments.

Let $\bar{x}_s$ be the distance of C.G. of simply supported B.M. dia. from A



$$\bar{x}_s = \frac{l+a}{3}$$

$$M_A + 2M_B = \frac{-6A_s\bar{x}_s}{l^2}$$

$$= \frac{-6}{l^2}\left(\frac{1}{2} \times \frac{wab}{l} \times l\right)\left(\frac{l+a}{3}\right)$$

$$= \frac{wab(a+l)}{l^2} \longrightarrow (2)$$

Subtracting (1) from (2)

$$M_B = -\frac{wab}{l^2}(a+l) + \frac{wab}{l}$$

$$= -\frac{wab}{l^2}(a+l-l) = -\frac{wa^2b}{l^2}$$

from (1) $\Rightarrow$ $$M_A = -\frac{wab}{l} - M_B$$

$$= -\frac{wab}{l} + \frac{wa^2b}{l^2}$$

$$= \frac{wab}{l^2}(l-a)$$

$$M_A = -\frac{wab^2}{l^2}$$

Max. +ve B.M. will be under the load and max. -ve B.M. will be either at supports A or B depending on relative values of 'a' and 'b'

slope and deflection

At any section distant x from the end A, the B.M. is $M_x = \cancel{\frac{wbx}{l}}$ Free BM $-$ Fixed B.M.

$$EI\frac{d^2y}{dx^2} = \frac{wb}{l}x - \left[M_a + \frac{M_b - M_a}{l}x\right] - w(x-a)$$

$$M_a + \frac{M_b - M_a}{l}x = \frac{wab^2}{l^2} + \frac{\frac{wa^2b}{l^2} - \frac{wab^2}{l^2}}{l}x$$

$$= \frac{wab^2}{l^2} + \frac{wab}{l^3}(a-b)x$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = \frac{wb}{l}x - \frac{wab^2}{l^2} - \frac{wab}{l^3}(a-b)x \; \vdots - w(x-a)$$

$$EI\frac{d^2y}{dx^2} = \frac{wb}{l^3}(l^2 - a^2 + ab)x - \frac{wab^2}{l^2} \; \vdots - w(x-a)$$

$$EI\frac{d^2y}{dx^2} = \frac{wb}{l^3}(3ab + b^2)x - \frac{wab^2}{l^2} \; \vdots - w(x-a)$$

Integrating

$$EI\frac{dy}{dx} = \frac{wb^2(3a+b)x^2}{2l^3} - \frac{wab^2}{l^2}x + C_1 \; \vdots - \frac{w(x-a)^2}{2}$$

(slope eq.)

But at $x = 0$ $\quad \frac{dy}{dx} = 0 \quad \therefore C_1 = 0$

Integrating again $\quad EIy = \frac{wb^2(3a+b)x^3}{6l^3} - \frac{wab^2x^2}{2l^2} + C_2 - \frac{w(x-a)^3}{6}$

(Def. eq.)

At $x = 0$, $y = 0$

$\therefore C_2 = 0$

$$EIy = \frac{wb^2(3a+b)x^3}{6l^3} - \frac{wab^2}{2l^2}x^2 - \frac{w(x-a)^3}{6}$$

<u>Deflection under the load</u>

putting $x = a$ in the def. eq.

$$EIy_c = \frac{wb^2(3a+b)a^3}{6l^3} - \frac{wab^2a^2}{2l^2}$$

$$= \frac{wa^3b^2}{6l^3}(3a + b - 3l)$$

$$\boxed{y_c = \frac{-wa^3b^3}{3EIl^3}}$$

Maximum Deflection

let $a > b$

Max. deflection will occur between A and C.

For this condition equating the slope to zero.

We have

$$0 = \frac{wb^2(3a+b)x^2}{2l^3} - \frac{wab^2}{l^2}x$$

$$\therefore \boxed{x = \frac{2al}{3a+b}}$$

Substitute in the deflection eq. we get

$$EI\, y_{max} = \frac{wb^2(3a+b)}{6l^3}\left(\frac{2al}{3a+b}\right)^3 - \frac{wab^2}{2l^2}\left(\frac{2al}{3a+b}\right)^2$$

$$= -\frac{wb^2}{6l^3}\left(\frac{2al}{3a+b}\right)^2\left[3al - \frac{(3a+b)(2al)}{(3a+b)}\right]$$

$$= \frac{-wb^2}{6l^3}\cdot\frac{4a^3l^2}{(3a+b)^2}\cdot(al)$$

$$= -\frac{2}{3}\cdot\frac{wa^3b^2}{(3a+b)^2}$$

$$\boxed{y_{max} = -\frac{2}{3}\frac{wa^3b^2}{(3a+b)^2 EI}}$$

Points of contraflexure

For the point of contraflexure in AC

$$M = \frac{wb^2}{l^3}(3a+b)x - \frac{wab^2}{l^2} = 0$$

$$\therefore \boxed{x = \frac{al}{3a+b}}$$

For the point of contraflexure in BC

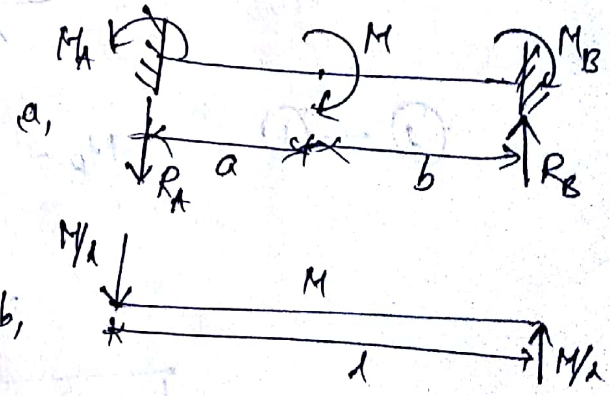
$$M = \frac{wb^2}{l^2}(3a+b)x - \frac{wab^2}{l^2} - w(ax-a) = 0$$

$$\frac{b^2}{l^2}(3a+b)x - \frac{ab^2}{l^2} - x + a = 0$$

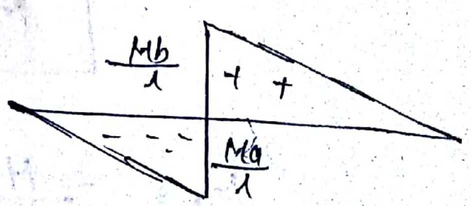
solving, we get $\boxed{x = l - \frac{bl}{3b+a}}$

(iv) Couple at distance 'a' from the left support

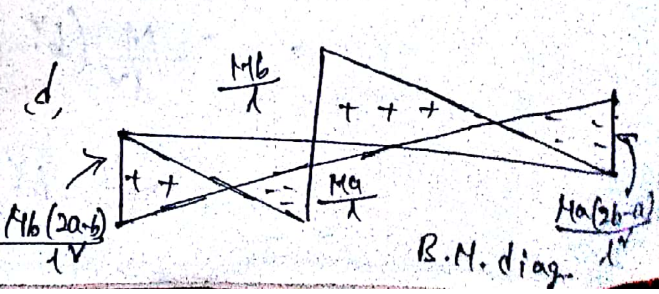
Consider the Beam as Simply supported beam with applied moments M. Reaction at each support be M/l



The B.M. diagram for the Simply supported beam will be as shown in fig.c,



Simply supported B.M.dia.



B.M. diag.

$$M_A + M_B = \frac{-2A_s}{l} = \frac{-2}{l}\left[-\frac{1}{2}\times\frac{Ma}{l}\times a + \frac{1}{2}\times\frac{Mb}{l}\times b\right]$$

$$= -\frac{M}{l^2}(b^2 - a^2) \longrightarrow (1)$$

$$M_A + 2M_B = \frac{-6A_s\bar{x}_s}{l^2}$$

$$= \frac{-6}{l^2}\left[-\frac{1}{2}\times\frac{Ma}{l}\times a\times\frac{2}{3}a + \frac{1}{2}\times\frac{Mb}{l}\times b\times\left(a+\frac{b}{3}\right)\right]$$

$$= \frac{-6}{l^2}\left[-\frac{Ma^3}{3l} + \frac{Mb^2}{2l}\times\frac{3a+b}{3}\right]$$

$$= \frac{-6}{l^2}\times\frac{M}{6l}\left[-2a^3 + 3ab^2 + b^3\right]$$

$$= -\frac{M}{l^3}(a+b)(b^2+2ab-2a^2)$$

$$= -\frac{M}{l^2}(b^2+2ab-2a^2) \longrightarrow (2)$$

(2) − (1) $\quad M_B = -\frac{M}{l^2}(b^2+2ab-2a^2-b^2+a^2)$

$$= \frac{-M}{l^2}(2ab-a^2)$$

$$\boxed{M_B = -\frac{Ma}{l^2}(2b-a)}$$

$$M_A = -\frac{M}{l^2}(b^2-a^2) + \frac{Ma}{l^2}(2b-a)$$

$$= -\frac{M}{l^2}(b^2-a^2-2ab+a^2)$$

$$= -\frac{M}{l^2}(b^2-2ab)$$

$$\boxed{\therefore M_A = -\frac{Mb}{l^2}(b-2a)}$$

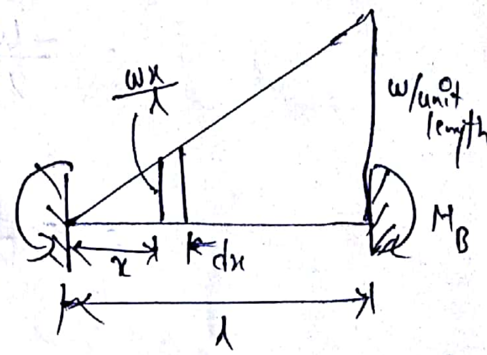
Take moments about B,

$$R_A \times l = \frac{Mb}{l^2}(2a-b) + \frac{Ma}{l^2}(2b-a) + M$$

$$= \frac{M}{l^2}(l^2 + 4ab - a^2 - b^2)$$

$$\boxed{R_A = \frac{6Mab}{l^3}}$$

V. <u>Uniformly varying load</u>



Consider a strip of width δx at distance x from support A. Intensity of loading at this section is $\frac{wx}{l}$

weight of elementary strip = $\frac{wx}{l}dx$

Fixed end moment δM_A and δM_B due to this elementary weight $\frac{wx}{l}dx$

$$\delta M_A = -\left(\frac{wx}{l}\right) \times \delta x \times \frac{x(l-x)^2}{l^2}$$

$$\delta M_B = -\left(\frac{wx}{l}\right) \times \delta x \times \frac{x^2(l-x)}{l^2}$$

$$M_A = -\int_0^l \frac{wx}{l}\,dx\,\frac{x(l-x)^2}{l^2}$$

$$= -\frac{w}{l^3}\int_0^l x^2(l^2-2lx+x^2)\,dx$$

$$= -\frac{w}{l^3}\left[\frac{l^2x^3}{3}-\frac{2lx^4}{4}+\frac{x^5}{5}\right]_0^l$$

$$= \frac{-w}{l^3}\left[\frac{l^5}{3}-\frac{2l^5}{4}+\frac{l^5}{5}\right]$$

$$= \cancel{\frac{-w}{l^3}\left[\frac{4l^5-6l^5+3l^5}{12}\right]}$$

$$= \frac{-wl^2}{30}$$

$$M_B = -\int_0^l \left(\frac{wx}{l}\right).dx \times \frac{x^2(l-x)}{l^2}$$

$$= \frac{-w}{l^3}\int_0^l (lx^3-x^4)\,dx = \frac{-w}{l^3}\left[\frac{lx^4}{4}-\frac{x^5}{5}\right]_0^l$$

$$= \frac{-w}{l^3}\left[\frac{l^5}{4}-\frac{l^5}{5}\right]$$

$$= -\frac{wl^2}{20}$$

<u>Reactions</u>

$$\Sigma M_B = 0 \rightarrow \quad R_A l - \frac{wl^2}{30} + \frac{wl^2}{20} - \frac{wl^2}{6} = 0$$

$$\boxed{R_A = \frac{3wl}{20}}$$

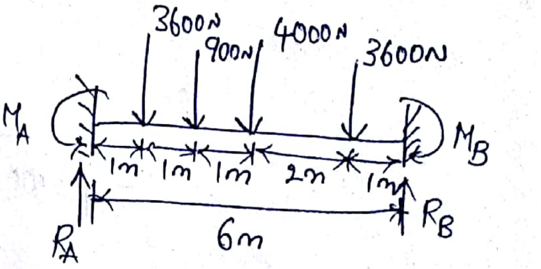
$$R_B = \frac{wl}{2} - \frac{3wl}{20}$$

$$\boxed{R_B = \frac{7wl}{20}}$$

Numerical problems on fixed Beams

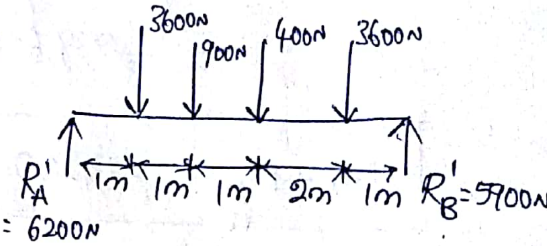
Ex. 1. Find the fixed end moments and plot the S.F. and B.M. diagrams for the beam loaded as shown in fig.(a),

(a)



Sol: Simply supported B.M. diagram is shown in fig.(b),

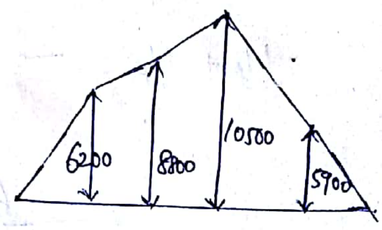
(b)



Reactions (R_A' and R_B') of the simply supported beam

Refer fig.(b),

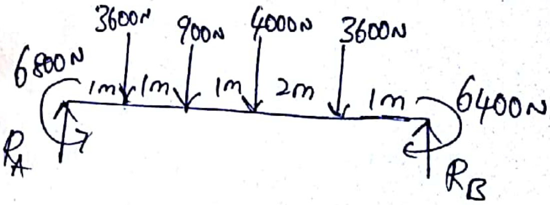
(c)



$\Sigma V = 0 \Rightarrow R_A' + R_B' = 3600 + 900 + 4000 + 3600$

$= 12,100 N$

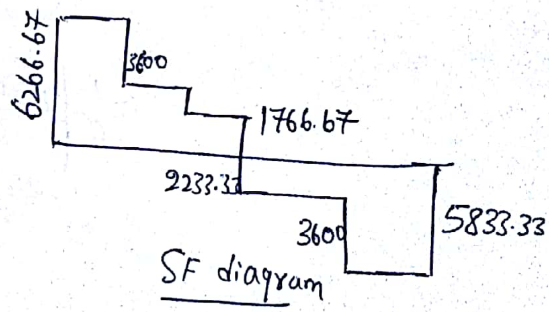
(d)



$\Sigma M_A = 0 \Rightarrow$

$R_B' \times 6 = 3600 \times 5 + 400 \times 3 + 900 \times 2 + 3600 \times 1$

(e)



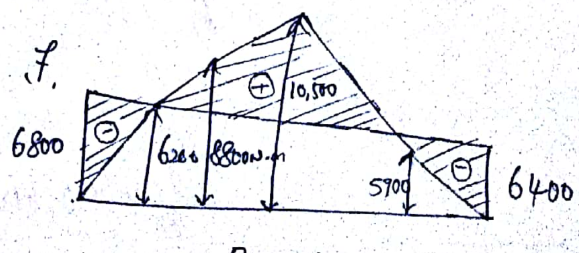
SF diagram

$$\boxed{R_B' = 5900 N}$$

$R_A' = 12,100 - 5900$

$$\boxed{R_A' = 6200 N}$$

(f)



B.M. diagram

$$M_A + M_B = -\frac{2A_s}{l}$$

$$= \frac{-2}{6}\left[\frac{1}{2}\times 6200\times 1 + \frac{1}{2}(6200+8800)\times 1 + \frac{1}{2}(8800+10{,}500)\times 1 + \frac{1}{2}(10{,}500+5900)\times 2 + \frac{1}{2}(5900\times 1)\right]$$

$$= \frac{-1}{3}\left[3100+7500+9650+16{,}400+2950\right]$$

$$= \frac{-1}{3}\times 39{,}600$$

$$\boxed{M_A + M_B = -13{,}200} \longrightarrow (1)$$

$$M_A + 2M_B = -\frac{6A_s\bar{x}_s}{l^2}$$

$$= -\frac{6}{6^2}\left[\frac{1}{2}\times 6200\times\frac{2}{3}(1) + 6200\times 1\times\frac{3}{2} + \frac{1}{2}\times 2600\left(1+\frac{2}{3}\right) + 8800\times\frac{5}{2} + \frac{1}{2}\times 1700 * \left(2+\frac{2}{3}\right) + 5900\times 2\times 4 + \frac{1}{2}\times 2(10{,}500-5900)\left(3+\frac{2}{3}\right) + \frac{1}{2}\times 5900\times\left(5+\frac{1}{3}\right)\right]$$

$$= \frac{-1}{6}\left[\frac{6200}{3} + 9300 + 1300 * \frac{5}{3} + 22{,}000 + 850\times\frac{8}{3} + 47{,}200 + 4600\times\frac{11}{3} + 5900\times\frac{8}{3}\right]$$

$$= \frac{-1}{6}\times\frac{1}{3}\left[6200 + 27{,}900 + 6500 + 66{,}000 + 6800 + 141{,}600 + 50{,}600 + 47200\right]$$

$$= \frac{-1}{18}\times 352{,}800$$

$$\boxed{M_A + 2M_B = -19{,}600} \longrightarrow (2)$$

(2) − (1) gives $M_B = -6400$ N-m

from (1) → $M_A = -6800$ Nm

Taking moments about A (fig d.)

$$R_B \times 6 + 6800 - 6400 - 3600 \times 1 - 3600 \times 5 - 900 \times 2 - 4000 \times 3 = 0$$

$$\boxed{R_B = 5833.33\text{N}}$$

$$\boxed{\therefore R_A = 6266.67\text{N}}$$

SF. and B.M. diagrams are shown in fig e. and fig f.

Max. −ve BM = −6800N

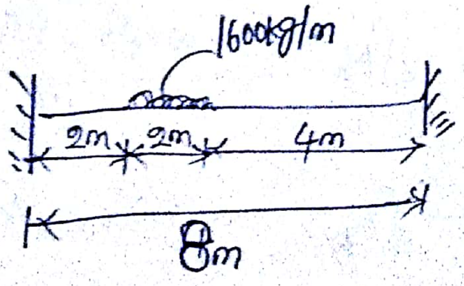
Max. +ve B.M at centre of span

$$= 10{,}500 - \frac{6800 + 6400}{2}$$

$$= 10{,}500 - 6600$$

$$= 3900 \text{ N-m}$$

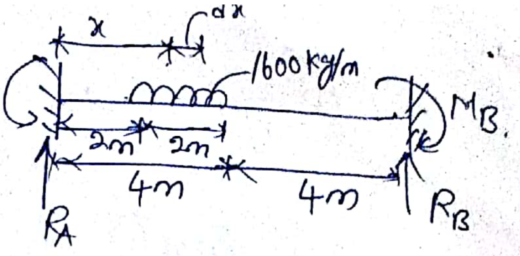
Ex-2: Find the fixed end moments and plot the B.M. diagram for the beam loaded as shown in fig a,



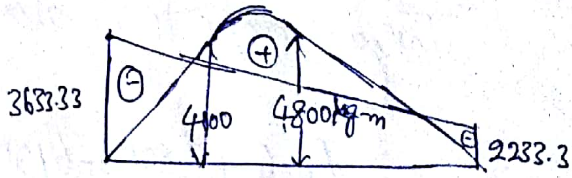
.01

Consider a small width δx of the Beam in loaded portion at distance 'x' from A

fig a,



b,



load on this strip = $w \times \delta x$

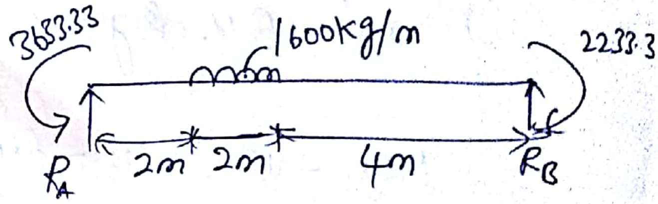
$= \cancel{16\delta x}$

$= 1600\delta x$

c,

Fixed end moments M_A and M_B due to a concentrated load at distance 'a' and 'b' from A and B respectively are given by

d,



$$M_A = \frac{-wab^2}{l^2}$$

$$M_B = \frac{-wa^2b}{l^2}$$

Due to elementary load $w \times \delta x$, fixed end moments δM_A and δM_B are given by

$$\delta M_A = \frac{-w \times \delta x \times x(l-x)^2}{l^2}$$

$$\delta M_B = \frac{-w \times \delta x \times x^2(l-x)}{l^2}$$

$$M_A = -\int_2^4 \frac{1600\,dx \times x(8-x)}{8^2}$$

$$= -\int_2^4 \frac{1600}{64}(64x - 16x^2 + x^3)\,dx$$

$$= -25\left(64\frac{x^2}{2} - \frac{16x^3}{3} + \frac{x^4}{4}\right)_2^4$$

$$= -25\left[32(16-4) - \frac{16}{3}(64-8) + \frac{1}{4}(256-16)\right]$$

$$= -25\left(384 - \frac{896}{3} + 60\right)$$

$$= \frac{-25 \times 436}{3} = -3633.33\,N\text{-}m$$

$$M_B = -\int_2^4 \frac{1600\,dx}{8^2} \times x^2(8-x)$$

$$= -25\int_2^4 (8x^2 - x^3)\,dx$$

$$= -25\left[8\frac{x^3}{3} - \frac{x^4}{4}\right]_2^4$$

$$= -25\left[\frac{8}{3}(64-8) - \frac{1}{4}(256-16)\right]$$

$$= -2233.3\,N\text{-}m$$

Taking moments about A (Ref. fig d.)

$$R_B \times 8 + 3633.33 - 2233.3 - 2 \times 1600 \times 3 = 0$$

$$\boxed{R_B = 1025 N}$$

$$R_A = 1600 \times 2 - 1025$$

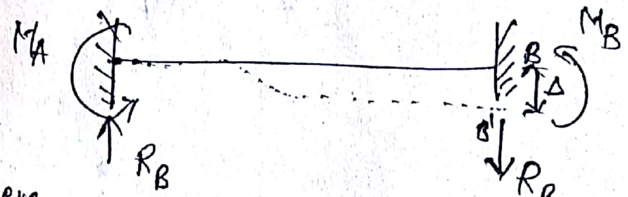
$$= 3200 - 1025$$

$$\boxed{R_A = 2175 N}$$

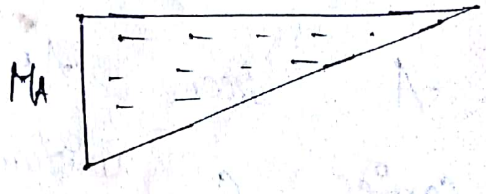
SF and BM. diagrams are shown in fig b and c

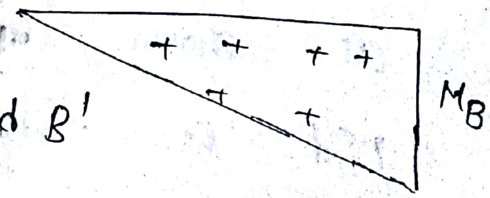
Effect of Sinking of Support

In simply supported beams, if one of the supports settles there is no effect on the moments and shears in the beam. However, in fixed beams if one of the supports settles or rotates relatively to other, there are moments developed at supports. These moments induce shear forces in the beams.



Consider a fixed beam AB as shown in fig. a, where the support B sinks by 'Δ' with respect to A and takes a position B'.





As the slopes at A and B' will be zero the total area of $\frac{M}{EI}$ diag. will b zero. Thus the fixing moments at two supports will be equal and opposite. Let M_A and M_B be the fixing moments at two supports.

$$M_A = -M_B$$

The intercept on the vertical by the two tangents drawn at A and B will be 'Δ'.

$$\frac{\left(\frac{1}{2}\times M_A\times l\right)\left(\frac{2}{3}l\right)+\left(\frac{1}{2}\times M_B\times l\right)\left(\frac{l}{3}\right)}{EI} = -\Delta$$

$$\frac{M_A l^2}{3}+\frac{M_B l^2}{6} = -EI\Delta$$

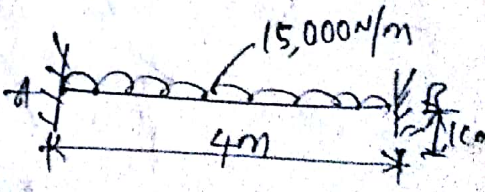
$$2M_A + M_B = \frac{-6EI\Delta}{l^2}$$

$M_A = -M_B$

$$\boxed{\therefore\ M_A = \frac{-6EI\Delta}{l^2} \qquad M_B = \frac{6EI\Delta}{l^2}}$$

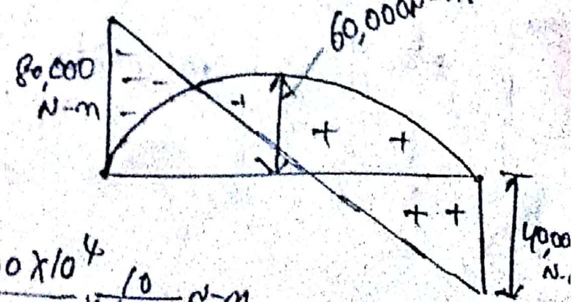
Ex

A beam AB span 4m fixed at A and B carries a uniformly distributed load of 15,000 N/m as shown in fig a. The support B sinks by 1cm. Find the fixed end moments and draw the Bending moment diagram for the beam. E = 2×10^5 N/mm^2 I = 8000 cm^4



$$M_A = M_B = \frac{-wl^2}{12} = \frac{-15{,}000\times4\times4}{12}$$

$= -20{,}000$ N-m



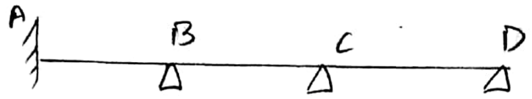
Due to settlement of support B

$$M_A = -M_B = \frac{-6EI\Delta}{l^2} = \frac{-6\times2\times10^5\times8000\times10^4}{4000^2}\times\frac{10}{1000}\text{ N-m}$$

$= -60{,}000$ N-m

UNIT-3

Continuous Beams



Internal support develops moments both sides the support

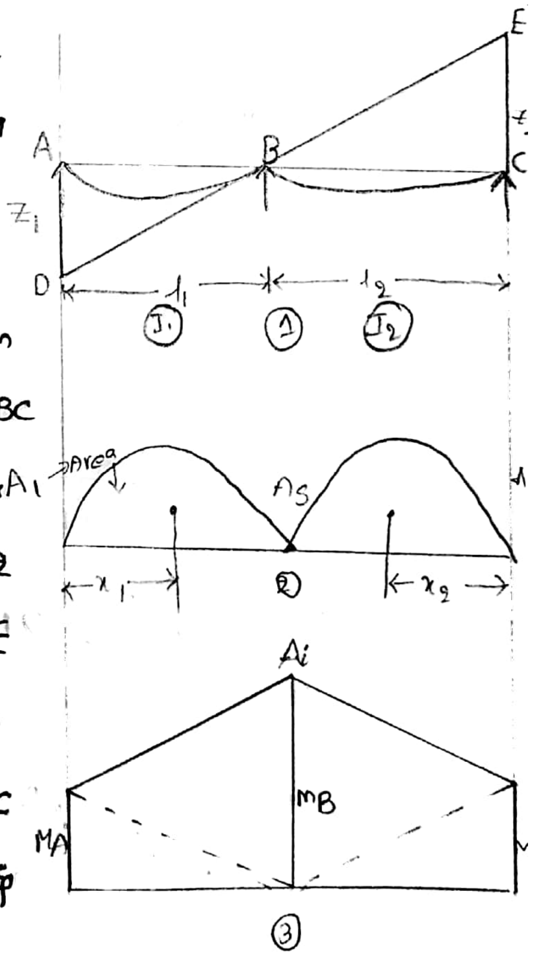
Assume slope 'θ_B'.

A beam which is supported on more than two supports is called continuous beam. Such beams when loaded deflect in the form of a curve such that at the intermediate supports the slopes of the elastic curve for the two spans will be same. At the intermediate support there will be B.M.

Clapeyrons theorem: (3-moment theorem).

consider two spans AB & BC of length l_1 & l_2.

Let I_1 & I_2 be the Moment of interia of spans AB & BC respectively. Let M_A, M_B, M_C be the moments of support at A, B, C respectively. Let A_1, A_2 area of simply supported bending moment diagram for the giving loading. considering AB & BC are S.S. Let x_1 and x_2 be the distance of center of gravity of areas A_1 & A_2 from A & C respectively. Draw D, B, E tangent to the elastic curve at middle support B' cutting the verticals at A & C in D & E respectively. Let $AD = Z_1$ & $CE = Z_2$ is above the elastic curve and therefore will be negative (Z_2).



z_1 : deflection of support 'A'

$$z_1 = \frac{A\bar{x}}{EI}$$

$$= \frac{A_1x_1 + \frac{1}{2} \times M_A \times l_1 \times \frac{l_1}{3} + \left(\frac{1}{2} M_B l_1\right)\left(\frac{2l_1}{3}\right)}{EI}$$

$$z_1 = \frac{A_1x_1 + \frac{M_A l_1^2}{6} + \frac{2M_B l_1^2}{6}}{EI_2}$$

$$z_2 = -\frac{A\bar{x}}{EI}$$

$$= \frac{A_2x_2 + \left(\frac{1}{2}M_C l_2\right)\left(\frac{l_2}{3}\right) + \left(\frac{1}{2}M_B l_2\right)\left(\frac{2l_2}{3}\right)}{EI_2}$$

$$z_2 = -\frac{A_2x_2 + \frac{M_C l_2^2}{6} + \frac{2M_B l_2^2}{6}}{EI_2}$$

$$\tan\theta_B = \frac{z_1}{l_1} = \frac{z_2}{l_2}$$

$$\therefore \frac{z_1}{l_1} = \frac{z_2}{l_2}$$

$$\frac{A_1x_1 + \frac{M_A l_1^2}{6} + \frac{2M_B l_1^2}{6}}{\cancel{E}I_1 l_1} = -\frac{A_2x_2 + \frac{M_C l_2^2}{6} + \frac{2M_B l_2^2}{6}}{\cancel{E}I_2 l_2}$$

$$\frac{A_1x_1}{I_1 l_1} + \frac{M_A l_1^2}{6I_1 l_1} + \frac{2M_B l_1^2}{6I_1 l_1} = -\frac{A_2x_2}{I_2 l_2} - \frac{M_C l_2^2}{6I_2 l_2} - \frac{2M_B l_2^2}{6I_2 l_2}$$

$$\frac{M_A l_1^2}{6I_1 l_1} + \frac{2M_B}{6}\left(\frac{l_1^2}{I_1 l_1} + \frac{l_2^2}{I_2 l_2}\right) + \frac{M_C l_2^2}{6I_2 l_2} = -\frac{A_1x_1}{I_1 l_1} - \frac{A_2x_2}{I_2 l_2}$$

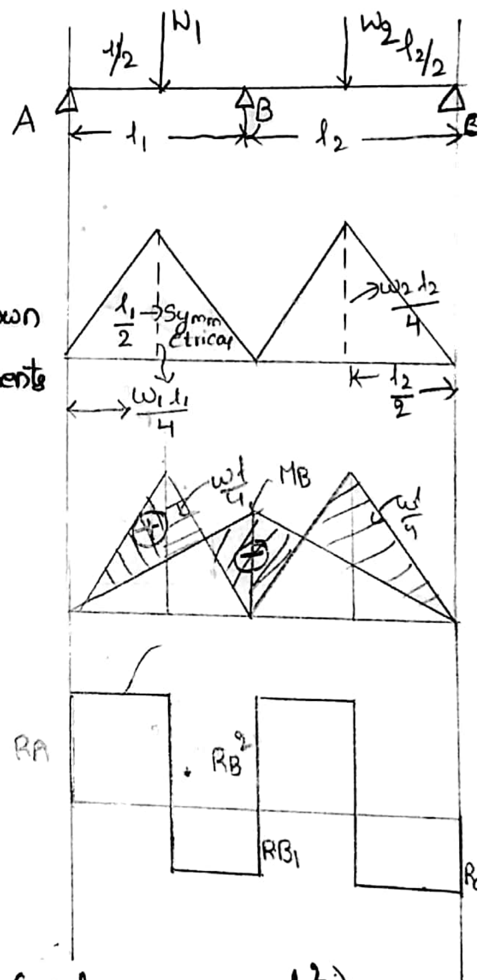
Multiply with '6'

$$* \boxed{M_A\left(\frac{l_1}{I_1}\right)+2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right)+M_C\left(\frac{l_2}{I_2}\right)=-6\left(\frac{A_1x_1}{I_1l_1}+\frac{A_2x_2}{I_2l_2}\right)}$$

Here $I_1 = I_2 = I$

$$\therefore \boxed{M_A(l_1)+2M_B(l_1+l_2)+M_C(l_2) = -6\left[\left(\frac{A_1x_1}{l_1}\right)+\left(\frac{A_2x_2}{l_2}\right)\right]}$$

* Analyse the continuous beam as shown in fig.(i) If $w_1 = W_2$ and (ii) $l_1 = l_2 = l$. Then find the moment at B. (M, R, S



let M_A, M_B, M_C be the support moments at A, B, & C respectively.

Here $M_A = M_C$ zero because simply supported. S.S Bending moment dia. shown in fig (2) clapeyrons theorem of 3 moments

(i)

$$A_1 = \frac{w_1 l_1}{8 l_1}$$

$$A_2 = \frac{w_2 l_2^2}{8 \times l_2}$$

$$x_1 = \frac{l_1}{2} \; ; \quad x_2 = \frac{l_2}{2}$$

$$0(l_1) + 2M_B(l_1+l_2) + 0(l_2) = -6\left(\frac{w_1 l_1}{8 \times l_1} \times \frac{l_1}{2} + \frac{w_2 l_2^2}{8 \times l_2} \times \frac{l_2}{2}\right)$$

$$2M_B(l_1+l_2) = -6 \times \frac{1}{16}(w_1 l_1^2 + w_2 l_2^2)$$

$$M_B = -\frac{3}{16}\left(\frac{w_1 l_1^2 + w_2 l_2^2}{l_1+l_2}\right)$$

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(ii) $w_1 = w_2 = w \quad ; \quad l_1 = l_2 = l$

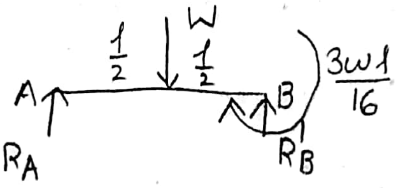
$$M_B = \frac{-3}{16}\left(\frac{wl^2 + wl^2}{l+l}\right)$$

$$= \frac{-3}{16} \times \frac{\cancel{2}wl^{\cancel{2}}}{\cancel{2l}}$$

$$M_B = \frac{-3}{16} wl$$

Reactions:-

Beam 'AB'



$$\Sigma V = 0 \Rightarrow R_A + R'_B = W$$

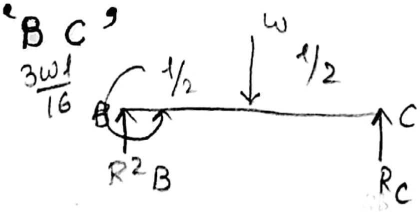
$$\Sigma M_A = 0 \Rightarrow -R'_B \times l + \frac{3wl}{16} + \frac{wl}{2} = 0$$

$$R'_B = \frac{3w}{16} + \frac{w}{2}$$

$$\boxed{R'_B = \frac{11w}{16}}$$

$$R_A = \frac{5w}{16}$$

Beam 'BC'



$$\Sigma V = 0 \Rightarrow RC + R^2B = w$$

$$\Sigma M_B = 0 \Rightarrow -R_C \times l - \frac{3wl}{16} + \frac{wl}{2} = 0$$

$$+R_C\, l = \frac{wl}{2} - \frac{3wl}{16}$$

$$R_C = \frac{8w - 3w}{16}$$

$$\boxed{R_C = \frac{5w}{16}}$$

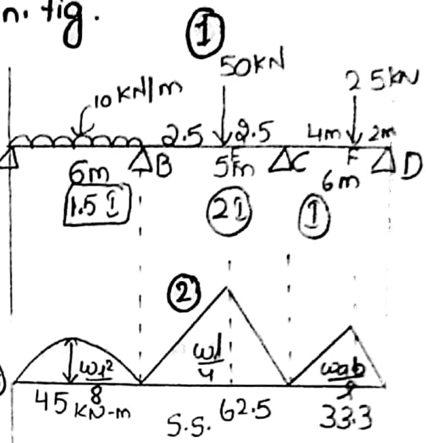
$$\boxed{R_B^2 = \frac{11w}{16}}$$

$$\therefore R_B = R_B^1 + R_B^2$$

$$= \frac{11w}{16} + \frac{11w}{16}$$

$$\boxed{R_B = \frac{22w}{16}}$$

Analyse the continuous beam as shown in fig. (Reac, Mom)



Let M_A, M_B, M_C and M_D be the support moments and R_A, R_B, R_C & R_D be the support reactions of the given beam.

Here $M_A = M_D = 0$ because ($M_B = M_C$ unknowns) simply supported. S.S Bending moment diagram for the given beam shown in fig 2

(In this 2 unknowns (M_B, M_C). so, calprons theorm 2 times). Symm $= 3(\bar{x}_1)$ $\bar{x}_2 = 2.5$

Applying calprons theorem for this spans A, B and BC.

$$M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_C\left(\frac{l_2}{I_2}\right) = -6\left(\frac{A_1\bar{x}_1}{I_1 l_1} + \frac{A_2 x_2}{I_2 l_2}\right)$$

Here $M_A = 0$ $\quad l_1 = 6$ $\quad A_1$ = Area of BMD 1st $= \frac{2}{3}bh = 180\,m^2$

$M_B = ?$ $\quad l_2 = 5$

$M_C = ?$

$$A_2 = \frac{1}{2} \times 5 \times 62.5 = 156.25\,m^2$$

$x_1 = 3$; $x_2 = 2.5$; $I_1 = 1.5I$; $I_2 = 2I$

$$0\left(\frac{6}{1.5I}\right) + 2M_B\left(\frac{6}{1.5I_1} + \frac{5}{2I}\right) + M_C\left(\frac{5}{2I}\right) = -6\left(\frac{180 \times 3}{1.5I_1 \times 6}\right) + \left(\frac{156.25 \times 2.5}{2I \times 5}\right)$$

$0 + 2M_B(6.5) + 2.5\,M_C = -594.37$

B.o.t multiply 2.5

$5.2\,M_B + M_C = -237.75 \rightarrow$ ① Equation

Applying calprons theorem of 3 moments for the spans

BC & CD

Here

$M_A = M_B$; $M_B = M_C$; $M_C = M_D = 0$

$l_1 = 5m$; $l_2 = 6m$; $I_1 = 2I$; $I_2 = I$

$A_1 = 156.25\,m^2$; $A_2 = \frac{1}{2} \times 6 \times 33.33 = 99.99\,m^2$

$x_1 = 2.5\,m$; $x_2 = \frac{l+b}{3} = \frac{6+2}{3} = 2.67\,m.$

$$M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left(\frac{l_1}{I_1} + \frac{l_2}{I_2}\right) + M_C\left(\frac{l_2}{I_2}\right) = -6\left(\frac{A_1 x_1}{I_1 l_1} + \frac{A_2 x_2}{I_2 l_2}\right)$$

$$M_B\left(\frac{5}{2I}\right) + 2M_C\left(\frac{5}{2I} + \frac{6}{I}\right) + 0\left(\frac{6}{I}\right) = -6\left(\frac{156.25 \times 2.5}{2I \times 5} + \frac{100 \times 2.67}{I \times 6}\right)$$

$2.5\,M_B + 17\,M_C = \cancel{-18} \,\, -501.375 \rightarrow$ ② Equation

Solving (1) & (2) Equations

① × 17

$$88.4\,M_B + \cancel{17 M_C} = -4041.75$$

$$② \Rightarrow 2.5\,M_B + \cancel{17 M_C} = -501.375$$

(+)

$$85.9\,M_B = -3540.375$$

$$M_B = -41.215 \text{ KN-m}$$

Substitute 'M_B' in Eq ②

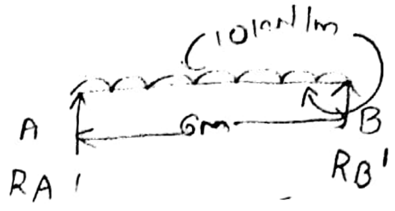
$$2.5 \times -41.215 + 17\,M_C = -501.375$$

$$17\,M_C = -501.375 + 103.0375$$

$$M_C = 23.43 \text{ KN-m}$$

1

Reactions:-



$\Sigma V = 0$

$R_A + R_B' = 10 \times 6 = 60$

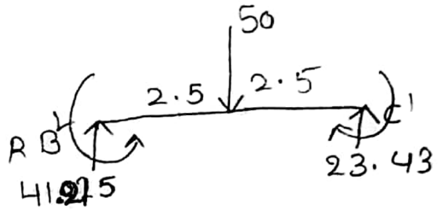
$\Sigma M = 0$

$- R_B' \times 6 + 41.25 + 10 \times 6 \times \frac{6}{2} = 0$

$R_B' = 36.86 \text{ KN}$

$R_A = 60 - 36.86$

$= 23.14 \text{ KN}.$



Beam BC

$R_C + R_B^2 = 50$

$\Sigma M = 0$

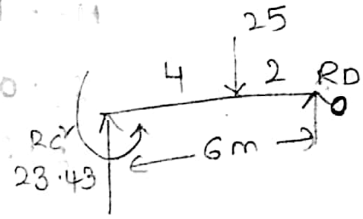
$- R_C' \times 5 + 23.43 - 41.215 + 50 \times 2.5 = 0$

$R_C' = 21.43 \text{ KN}$

$R_B^2 + R_C' = 50$

$R_B^2 = 28.5.$

Beam CD



$R_C^2 + R_D = 25$

$\Sigma M_C = 0$

$- R_D \times 6 + 25 \times 4 - 23.43 = 0$

$R_D = 12.76$

$R_C^2 + R_D = 25$

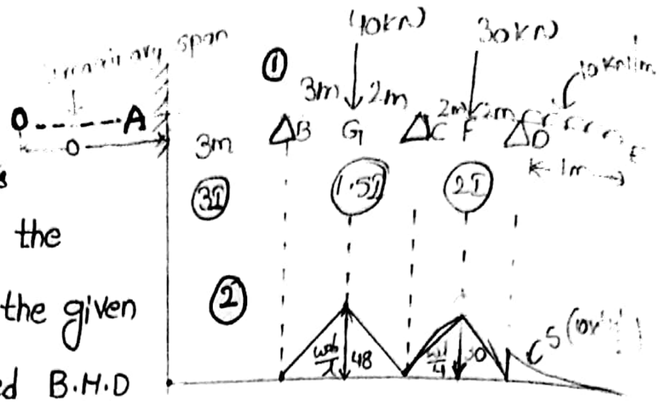
$R_C^2 = 12.24.$

$\therefore$ Reactions at the supports:-

$R_A = 23.14 \text{ KN}$ $\quad R_B = R_B' + R_B^2 = 65.36 \text{ KN}$

$R_C = R_C' + R_C^2 = 33.67 \text{ KN}$ $\quad R_D = 12.76 \text{ KN}$

* Analyse the continuous beam as shown in fig



Let M_A, M_B, M_C, & M_D be the support moments and R_A, R_B, R_C, & R_D be the support reactions of the given beam. Simply supported B.M.D is shown in fig.

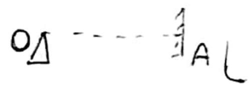
Here
$$\boxed{\begin{aligned} M_D &= -10\times1\times\tfrac{1}{2} \\ &= -5\ \text{kN-m}\end{aligned}}$$

Calpron's theorem of 3 moments

$$M_A\left(\frac{l_1}{I_1}\right)+2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right)+M_C\left(\frac{l_2}{I_2}\right)=-6\left(\frac{A_1x_1}{I_1l_1}+\frac{A_2x_2}{I_2l_2}\right)$$

Applying theorem of 3-moments for Imaginary span 'O A' & 'AB'

Here $M_A = 0$



$M_B = M_A$

$M_C = M_B$

$l_1 = 0$; $I_1 = 0$; $A_1 = 0$; $x_1 = 0$

$l_2 = 3$; $I_2 = 3I$; $A_2 = 0$; $x_2 = 0$

$$0+2M_A\left(0+\frac{\cancel{3}}{\cancel{3}I}\right)+M_B\left(\frac{\cancel{3}}{\cancel{3}I}\right)=-6\left(\frac{0\times0}{0\times0}+\frac{0\times0}{3I\times3}\right)$$

$$2M_A+M_B=0 \longrightarrow ①\ \text{Eq.}\quad \left(M_A=\frac{-M_B}{2}\right)$$

Applying calperons theorem for the spans 'AB' & 'BC'

Here $M_A = M_A$ $l_1 = 3$ $I_2 = 1.5I$

$M_B = M_B$ $l_2 = 5$ $A_1 = 0$

$M_C = M_C$ $I_1 = 3I$ $A_2 = \frac{1}{2}\times5\times48$

$x_1 = 0$; $x_2 = \frac{l+b}{3} = \frac{5+2}{3} = 2.33$ $= 120\,m^2$

$$M_A\left(\frac{\cancel{3}}{3\cancel{I}}\right)+2M_B\left(\frac{\cancel{3}}{3\cancel{I}}+\frac{5}{1.5I}\right)+M_C\left(\frac{5}{1.5I}\right)=-6\left(\frac{0\times 0}{3I\times 3}\right)+\left(\frac{120\times 2.}{1.5I\times 5}\right)$$

$\cancel{\div M_B}$.

$M_A + 8.67M_B + 3.33M_C = -220.8$

$\cancel{\frac{M_B}{2}}$ $0.5M_B + 8.67M_B + 3.33M_C = -220.8$

$8.17M_B + 3.33M_C = -220.8 \rightarrow$

Divide by 3.33

$2.45M_B + M_C = -66.30 \rightarrow$ 2 Equation

Applying calprons theorem for the span 'BC' & 'CD'

$M_A = M_B$; $l_1 = 5$; $I_1 = 1.5$; $l_2 = 4$; $I_2 = 2I$

$A_1 = 120m^2 = x_1 = \frac{l+a}{3} = 2.6$; $A_2 = \frac{1}{2}\times 30\times 4$ $x_2 = 2$

$$M_B\left(\frac{5}{1.5I}\right)+2M_C\left(\frac{5}{1.5I}+\frac{4\cancel{2}}{\cancel{2}I}\right)+M_D\left(\frac{4}{2I}\right)=-6\left(\frac{120\times 2.6}{1.5I\times 5}+\frac{60\times\cancel{2}}{\cancel{2}I\times 4}\right)$$

$3.33M_B + 10.67M_C + 2M_D = -339.6$

$3.33M_B + 10.67M_C = +10 - 339.6$ $\left(\begin{array}{l}2M_D\\=2\times -5\\=-10\end{array}\right)$

$3.33M_B + 10.67M_C = -329.6 \rightarrow$ 3 Eq

2 $\times$ 10.67.

$25.97M_B + 10.67M_C = -709.14$

3 − 2

$3.33M_B + \cancel{10.67M_C} = -329.6$

$\underset{(-)}{25.97M_B} \underset{(-)}{+\cancel{10.67M_C}} = \underset{(+)}{-709.14}$

$-22.6M_B = 379.5$

$\boxed{M_B = -16.79\ KN\text{-}m}$

Sub eq 2

$2.45M_B + M_C = -66.30$

$\boxed{M_C = -25.28\ KN\text{-}m}$

$$M_A\left(\frac{\cancel{3}}{3\cancel{I}}\right)+2M_B\left(\frac{\cancel{3}}{3\cancel{I}}+\frac{5}{1.5I}\right)+M_C\left(\frac{5}{1.5I}\right)=-6\left(\frac{0\times0}{3I\times3}\right)+\left(\frac{120\times2.}{1.5I\times5}\right.$$

$(\div M_B)$.

$$M_A+8.67M_B+3.33M_C=-220.8$$

$$\cancel{\frac{M_B}{2}}\;0.5M_B+8.67M_B+3.33M_C=-220.8$$

$$8.17M_B+3.33M_C=-220.8\rightarrow$$

Divide by 3.33

$$2.45M_B+M_C=-66.30\rightarrow (2)\text{ Equation}$$

Applying calprons theorem for the span 'BC' & 'CD'

$M_A=M_B$; $l_1=5$; $I_1=1.5$; $l_2=4$; $I_2=2I$

$A_1=120m^2=x_1=\frac{1+a}{3}=2.6$; $A_2=\frac{1}{2}\times30\times4$ $x_2=2$

$$M_B\left(\frac{5}{1.5I}\right)+2M_C\left(\frac{5}{1.5I}+\frac{4^2}{\cancel{2}I}\right)+M_D\left(\frac{4}{2I}\right)=-6\left(\frac{120\times2.6}{1.5I\times5}+\frac{60\times\cancel{2}}{\cancel{2}I\times4}\right)$$

$$3.33M_B+10.67M_C+2M_D=-339.6$$

$$3.33M_B+10.67M_C=+10-339.6$$

$$\left(\begin{matrix}2M_D\\=2\times-5\\=-10\end{matrix}\right)$$

$$3.33M_B+10.67M_C=-329.6\longrightarrow (3)\text{ Eq}$$

(2) × 10.67.

$$25.97M_B+10.67M_C=-709.14$$

(3) − (2)

$$3.33M_B+\cancel{10.67M_C}=-329.6$$

$$\underset{(-)}{25.97M_B}\underset{(-)}{+}\cancel{10.67M_C}=\underset{(+)}{-}709.14$$

$$-22.6M_B=379.5$$

$$\boxed{M_B=-16.79\text{ KN-m}}$$

Sub eq (2)

$$2.45M_B+M_C=-66.30$$

$$\boxed{M_C=-25.28\text{ KN-m}}$$

Sub eq ①

$2MA + MB = 0$

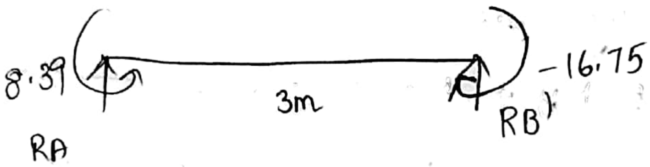
$2MA - 16.79 = 0$

$2MA = 16.79$

$\boxed{MA = 8.39\ kN\text{-}m}$

Reactions:-

AB



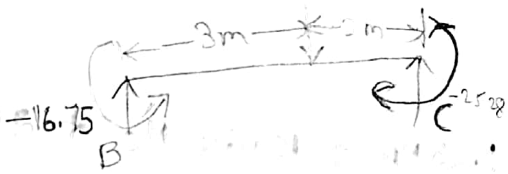
$\Sigma V = 0$

$RA + RB' = 0$

$-RB \times 3 + 16.75 - 8.39 = 0$

$\therefore RB' = 2.78\ kN$

$\boxed{RA = -2.78\ kN}$



$RB^2 + RC^1 = 40$

$-RB \times 5 - 16.75 + 25.28 = 0$

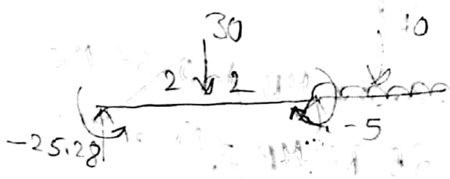
$RB^2 = 1.706\ kN$

$RC = 38.29\ kN$

$-RC \times 5 + 40 \times 3 - 16.75 + 25.28 = 0$

$RC' = 25.706\ kN$

$RB^2 = 14.294\ kN$



$\Sigma V = 0$

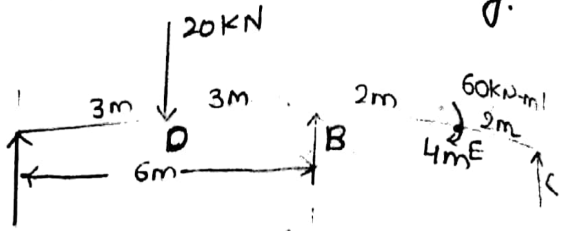
$RD + RC^{v} = 30$

$-RD \times 4 + 30 \times 2 - 25.28 + 5 = 0$

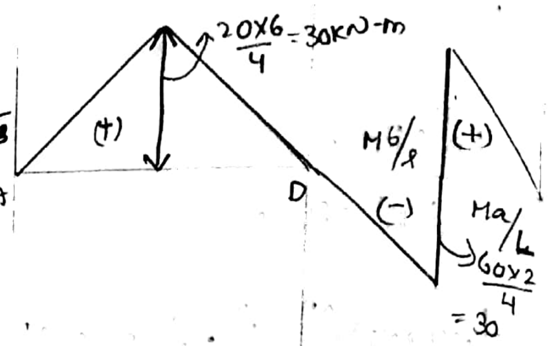
$\boxed{RD = 9.93}$; $RC^2 = 20.07$

$\boxed{RC = 45.768\ kN}$ $\boxed{RB = 17.074\ kN}$

Analyse the continuous beam as shown in fig.



Simply supported BMD is shown in fig (2). Let M_A, M_B & M_C be the moments at the supports and R_A & R_B, R_C be the reactions at the supports for the given beam



Here $M_A = M_C = 0$

Calpeyron's theorem

$$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = -6\left(\frac{A_1x_1}{l_1} + \frac{A_2x_2}{l_2}\right)$$

Here $M_A = 0$ $\quad l_1 = 6m$ $\quad A_1 = \frac{1}{2}\times30\times6 = 90m^2$

$m_B = M_B$ $\quad l_2 = 4m$

$M_C = 0$

$$A_2x_2 = \left(\frac{1}{\cancel{2}}\times30\times\cancel{2}\right)\left(\frac{2}{3}\times2\right) - \left(\frac{1}{\cancel{2}}\times30\times\cancel{2}\right)\left(2+\frac{1}{3}(2)\right)$$

$= 40 - 80$

$= -40\,m^3$; $\quad x_1 = 3m.$

$$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = -6\left(\frac{A_1x_1}{l_1} + \frac{A_2x_2}{l_2}\right)$$

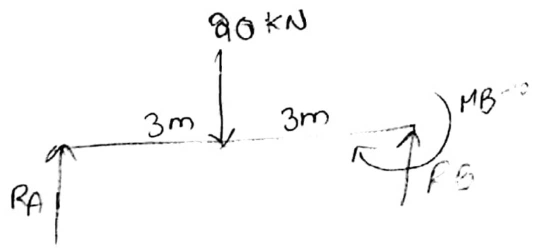
$$0 + 2M_B(6+4) + 0 = -6\left(\frac{90\times3}{6} - \frac{40}{4}\right)$$

$20M_B = -210$

$$\boxed{M_B = -10.5\ KN\text{-}m}$$

Reactions:

$AB \Rightarrow$ Beam



$\Sigma V = 0$

$R_A + R_B' = 20KN$

$\Sigma M_A = 0$

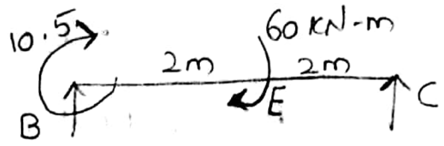
$-R_B'\times6 + 20\times3 + 10.5 = 20$

$R_B' = 11.75\ KN$

$R_A + R_B{}^1 = 20$ kN

$\boxed{R_A = 8.25 \text{ kN.}}$

Portion BC



$R_B{}^2 + R_C = 0$

$\Sigma M_A = 0$

$-R_C \times 4 + 60 - 10.5 = 0$

$\boxed{R_C = 12.37 \text{ kN}}$

$R_B{}^2 + R_C = 0$

$R_B{}^2 = -R_C$

$R_B{}^2 = -12.37$

$R_B = R_B{}^1 + R_B{}^2$

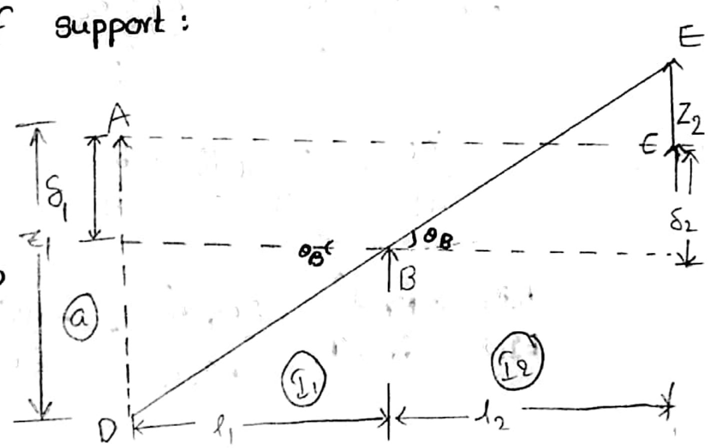
$\boxed{R_B = -0.62 \text{ kN}}$

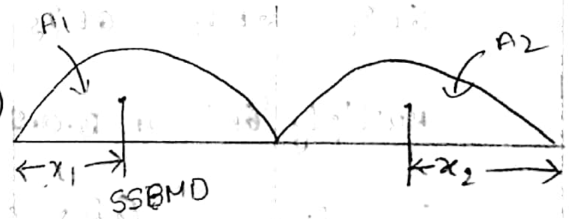
Effect of sinking of support :

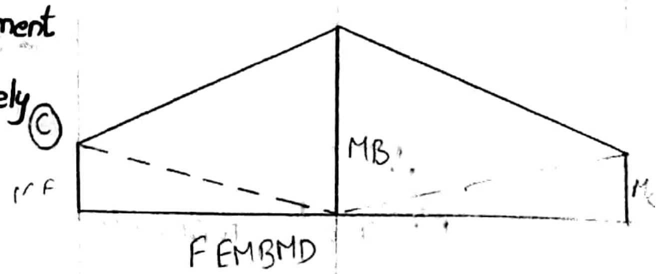
Let the level of middle Support 'B' be the 'δ_1' Below support A & δ_2 below Support c.

Draw tangent at B to the elastic curve (at B) cutting the verticals at A & C at D & E respectively as shown in fig (a).

Let the spans AB and BC be of length l_1 and l_2. And moment of inertia I_1 & I_2 respectively







$$Z_1 = \frac{A\bar{x}_1}{EI_1}$$

$$= \frac{A_1 x_1 + \left(\frac{1}{2} M_A l_1\right)\left(\frac{l_1}{3}\right) + \left(\frac{1}{2} M_B l_1\right)\left(\frac{2}{3} l_1\right)}{EI_1}$$

$$= \frac{A_1 x_1}{EI_1} + \frac{M_A l_1^2}{6EI} + \frac{2 M_B l_1^2}{6EI_1}$$

$$Z_2 = -\frac{A\bar{x}}{EI_2}$$

$$= -\frac{A_2 x_2 + \left(\frac{1}{2} M_C l_2\right)\left(\frac{l_2}{3}\right) + \left(\frac{1}{2} M_B l_2\right)\left(\frac{2}{3} l_2\right)}{EI_2}$$

$$= -\frac{A_2 x_2}{EI_2} - \frac{M_C l_2^2}{6EI_2} - \frac{2 M_B l_2^2}{6I_2 E}$$

$$\therefore \frac{Z_1 - \delta_1}{l_1} = \frac{Z_2 + \delta_2}{l_2}$$

$$\frac{Z_1}{l_1} = \frac{Z_2}{l_2} + \frac{\delta_2}{l_2} + \frac{\delta_1}{l_1}$$

$$\frac{A_1 x_1}{EI_1 l_1} + \frac{M_A l_1^2}{6I_1 l_1 E} + \frac{2 M_B l_1^2}{6EI_1 l_1} = -\frac{A_2 x_2}{EI_2 l_2} - \frac{M_C l_2^2}{6EI_2 l_2} - \frac{2 M_B l_2^2}{6EI_2 l_2} + \frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}$$

$$\frac{M_A l_1}{6EI_1} + \frac{2M_B l_1}{6EI_1} + \frac{2M_B l_2}{6EI_2} + \frac{M_C l_2}{6EI_2} = -\frac{A_1 \bar{x}_1}{EI_1 l_1} - \frac{A_2 \bar{x}_2}{EI_2 l_2} + \frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}$$

Multiply '6E' on B.O.S

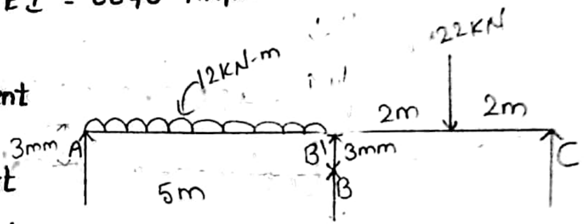
$$\boxed{M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left[\left(\frac{l_1}{I_1}\right) + \left(\frac{l_2}{I_2}\right)\right] + M_C\left(\frac{l_2}{I_2}\right) = -6\left(\frac{A_1\bar{x}_1}{I_1 l_1} + \frac{A_2\bar{x}_2}{I_2 l_2}\right) + 6E\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)}$$

Different moment of inertia

Incase $I_1 = I_2 = I$ (Same moment of inertia).

$$\boxed{M_A(l_1) + 2M_B(l_1 + l_2) + M_C(l_2) = -6\left(\frac{A_1 x_1}{l_1} + \frac{A_2 x_2}{l_2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)}$$

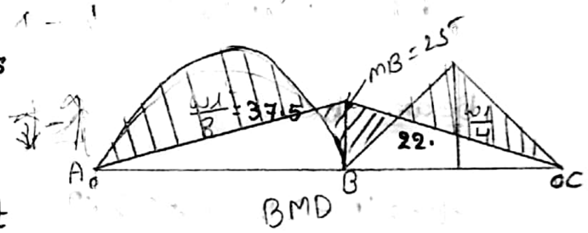
A two span continuous beam ABC rest on simply supports at A B C the span AB = 5m ; Span BC = 4m ; The span AB a uniformly distrubted load of 12 kN-m and span BC a central point load of 22 kN ; EI is constant for the whole beam. If the support B settles by 3mm. Find the support moments and reactions at all the supports using calperons : $EI = 6640\ KN/m^2$



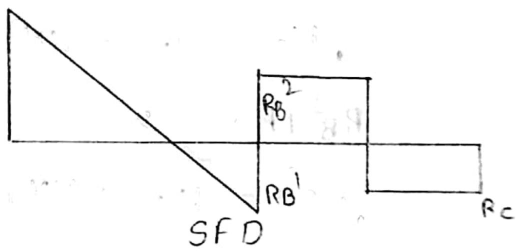
NOTE:- Sign convention for settlement

For left ~~Support~~ Span :- w.r.t left support right support is down then 'δ_1' is (+)

w.r.t left support is right support is up then 'δ_1' is (-)



For Right span :- w.r.t left support right is up then 'δ_2' is (+). w.r.t left support is right is down then 'δ_2' is (-)



Let M_A, M_B, M_C be the support moments, R_A, R_B, R_C be the support reactions for the given beam. S.S.B.M.D is shown in fig (2).

Here $M_A = M_C = 0$ (S.S) calpeyron's theorem of '3' moments

$$M_A(l_1) + 2M_B(l_1 + l_2) + M_C(l_2) = -6\left(\frac{A_1 x_1}{l_1} + \frac{A_2 x_2}{l_2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

Here Applying calpeyron's theorem AB & BC

Here $M_A = 0$ $A_1 = \frac{2}{3} bh = 125 m^2$ $EI = 6640\ KN\text{-}m^2$

$M_B = M_B$ $A_2 = \frac{1}{2} \times 4 \times 22$ $\delta_1 = +3mm \Rightarrow 0.003$

$M_C = 0$ $= 44 m^2$ $\delta_2 = +3mm$

$l_1 = 5$ $x_1 = 2.5 m$

$l_2 = 4$ $x_2 = 2m$

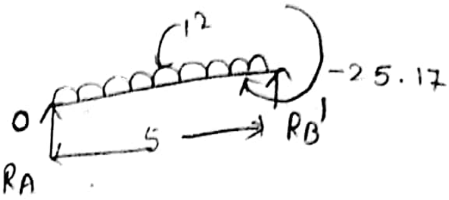
$$0(5) + 2M_B(5+4) + 0(12) = -6\left(\frac{125 \times 2.5}{5} + \frac{44 \times 2}{4}\right) + 6 \times 6640 \left(\frac{0.004}{5} + \frac{0.003}{4}\right)$$

$$0 + 2M_B(9) = 18M_B$$

$$\boxed{M_B = -25.17 \text{ KN-m}}$$

Reactions:-

Beam AB



$$R_A + R_B^1 = 12 \times 5 = 60$$

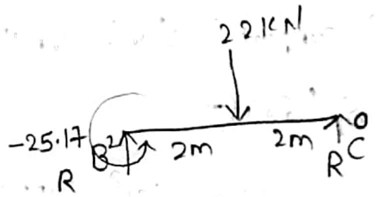
$$\Sigma M_A = 0$$

$$-R_B^1 \times 5 + 25.17 + 12 \times 5 \times \frac{5}{2} = 0$$

$$\boxed{R_B^1 = 35.03 \text{ KN}}$$

$$\boxed{R_A = 24.96 \text{ KN}}$$

Beam BC



$$-R_C \times 4 - 25.17 + 22 \times 2$$

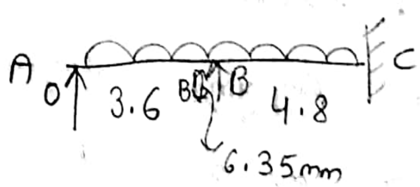
$$\boxed{R_C = 4.707 \text{ KN}}$$

$$R_B^2 + R_C = 22$$

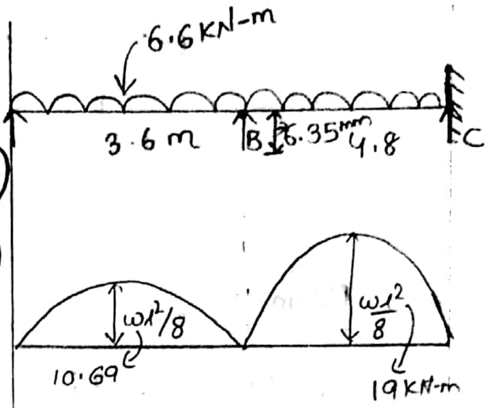
$$\boxed{R_B^2 = 17.29 \text{ KN}}$$

$$R_B = R_B^1 + R_B^2 = 52.32.$$

Analayse the continuous beam as shown in fig. It carries a udl of 6.6 KN-m on AB and BC. The Support B sinks by 6.35mm below A & C. Take EI = 6640 kN/m²



Applying calpyerons theorem for the spans AB & BC



$$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = -6\left(\frac{A_1x_1}{l_1} + \frac{A_2x_2}{l_2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

Here $M_A = 0$ $\quad A_1 = \frac{2}{3}bh$

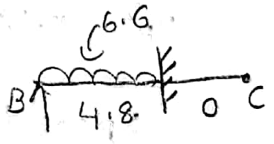
$M_B = M_B$ $\quad x_1 = 3.6/2$

$M_C = M_C$

$$0 + 2M_B(3.6+4.8) + M_C(4.8) = -6\left(\frac{\frac{2}{3}\times 10.69\times 3.6 \times \frac{3.6}{2}}{3.6} + \frac{A_2x_2}{l_2}\right) + 6EI\left(\frac{\delta_1}{l_1} + \frac{\delta_2}{l_2}\right)$$

$$0 + 2M_B(3.6+4.8) + M_C(4.8) = -6\left(\frac{46.18}{3.6} + \frac{\frac{2}{3}\times 19\times 4.8\times\left(\frac{4.8}{2}\right)}{4.8}\right) + 6EI\left(\frac{6.35}{1000\times 3.6} + \frac{6.35}{1000\times 4.8}\right)$$

$$16.8M_B + 4.8M_C = -136.66 \longrightarrow ① \text{ Eq.}$$



$M_B = M_A$; $M_C = M_B$; $M_C = 0$

$l_1 = 4.8$; $l_2 = 0$; $x_1 = 2.4$ $\quad A_1 = \frac{2}{3}bh$

$= \frac{2}{3}\times 4.8\times 19$

$A_2 = 0$; $x_2 = 0$; $\delta_2 = 0$; $\delta_1 = -6.35$ $\quad = 60.8$

$$M_B(4.8) + 2M_C(4.8+0) + 0(0) = -6\left[\frac{60.8\times 2.4}{4.8} + 0\right] + 6\times 6640\left(\frac{-6.35}{1000\times 4.8}\right)$$

$$4.8M_B + 9.6M_C = -6[30.4] + 6\times 6640(-1.32\times 10^{-3})$$

$$4.8M_B + 9.6M_C = -182.4 - 52.58$$

$$4.8M_B + 9.6M_C = -234.98 \longrightarrow ② \text{ Eq.}$$

$$① \times 2 \Longrightarrow 33.6M_B + 9.6M_C = -273.32$$

② − ①

$$4.8M_B + \cancel{9.6M_C} = -234.98$$

$$(-)\ 33.6M_B \overset{(-)}{+} \cancel{9.6M_C} = \underset{(+)}{-}273.32$$

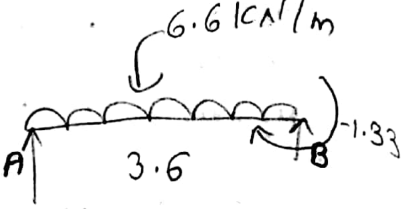
$$-28.8M_B = 38.34 \qquad \boxed{M_B = -1.33}$$

$$16.8 \times (-1.33) + 4.8 M_C = -136.66$$

$$\boxed{M_C = -23.81 \text{ KN-m}}$$

Reactions:-

Beam AB

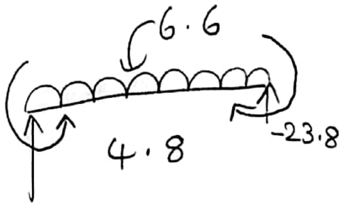


$R_A + R_B^1 = 23.76.$

$$-R_B^1 \times 3.6 + 1.33 + 6.6 \times 3.6 \times \frac{3.6}{2} = 0$$

$R_B^1 = 12.24$ KN. $\quad R_A + R_B^1 = 23.76$

$$\boxed{R_A = 11.52 \text{ kN}}$$



$R_B^2 + R_C = 6.6 \times 4.8 = 31.68$

$$-R_C \times 4.8 + 23.8 - 1.33 + 6.6 \times 4.8 \times \frac{4.8}{2} = 0$$

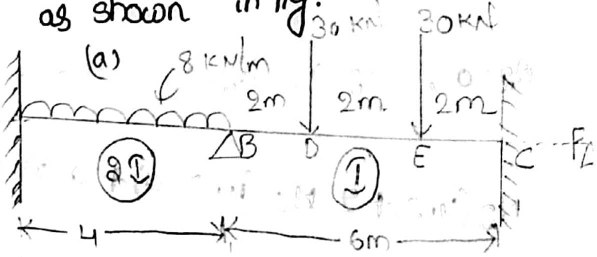
$$\boxed{R_C = 20.5 \text{ kN}}$$

$R_B^2 + R_C = 31.68$

$R_B^2 = 11.18$ KN

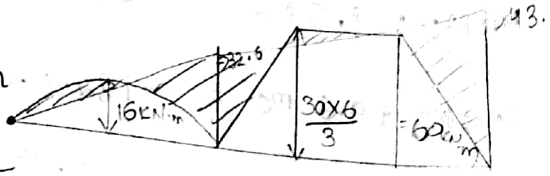
$$\boxed{R_B = R_B^1 + R_B^2 = 23.42 \text{ KN}}$$

Analyse the C.B as shown in fig. (a)



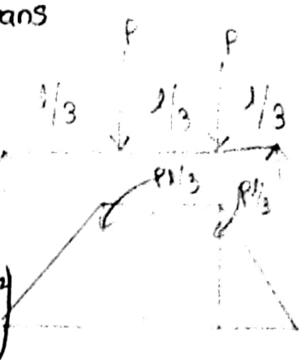
(b)

Let M_A, M_B & M_C be the support moments and R_A, R_B & R_C be the support reactions for the given beam. S.S. B.M.D is shown in fig(B)



Applying theorem of 3 moments for the spans

1. OA & AB   2. AB & BC   3. BC & CP



1. moment apply theorem OA & AB

$$M_A\left(\frac{l_1}{I_1}\right)+2M_B\left(\frac{l_1}{I_1}+\frac{l_2}{I_2}\right)+M_C\left(\frac{l_2}{I_2}\right)=-6\left(\frac{A_1x_1}{I_1l_1}+\frac{A_2x_2}{I_2l_2}\right)$$

OA & OB

$M_A=0$; $M_B=M_A$; $M_C=M_B$; $l_1=0$; $l_2=4m$; $I_1=0$; $I_2=2I$

$A_1=0$; $A_2=42.66$; $x_1=0$; $x_2=2m$.

$$\cancel{2}M_A\left(\frac{0}{0}+\frac{\cancel{4}^{2}}{\cancel{2}\cancel{I}}\right)+M_B\left(\frac{4^2}{2I}\right)=-6\left[\frac{0\times0}{0\times0}+\frac{42.66\times\cancel{2}}{2\cancel{I}\times\cancel{4}_{2}}\right]$$

$$2M_A+2+2M_B=-63.99$$

$$4M_A+2M_B=-64.$$

Divided by 2

$$2M_A+M_B=-32 \longrightarrow (1)\ \text{Equ.}$$

Applying theorem of 3 moments for the spans AB & BC

Here:-

$M_A=M_A$; $M_B=M_B$; $M_C=M_C$

$l_1=4m$; $l_2=6$; $I_1=2I$ $I_2=I$; $A_1=42.67$; $A_2=\frac{a+b}{2}(h)$

$=\frac{60}{2}(2+6)$

$=240\,m^2$

$x_1=2m$; $x_2=3m$;

$$M_A\left(\frac{4}{2I}\right)+2M_B\left(\frac{4^2}{2I}+\frac{6}{I}\right)+M_C\left(\frac{6}{I}\right)=-6\left[\frac{42.67\times\cancel{2}}{\cancel{2}I\times4}+\frac{240\times\cancel{3}}{I\times\cancel{6}_{2}}\right]$$

$$2M_A+16M_B+6M_C=-6\left[10.66+120\right]$$

$$2M_A+16M_B+6M_C=-784$$

Divide by '2'

$$M_A+8M_B+3M_C=-392 \longrightarrow (2)\ \text{Equ.}$$

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Applying theorem of 3 moments spans

BC & CP

$M_B = M_A$; $M_C = M_B$; $M_C = 0$

$l_1 = 6$; $l_2 = 0$; $A_1 = 240$; $A_2 = 0$; $x_1 = 3$; $x_2 = 0$

$I_1 = I$; $I_2 = 0$

$$M_B\left(\frac{6}{I}\right) + 2M_C\left(\frac{6}{I} + \frac{0}{0}\right) + 0\left[\frac{0}{0}\right] = -6\left[\frac{240 \times 3}{I \times 6}\right] + 0$$

$$6M_B + 12M_C = -720$$

Divide by 6

$$M_B + 2M_C = -120 \rightarrow (3) \text{ Equ.}$$

$$2M_A + M_B = -32 \rightarrow (1)$$

$$M_A + 8M_B + 3M_C = -392 \rightarrow (2)$$

$$M_B + 2M_C = -120 \rightarrow (3)$$

from (1) Equation

$$2M_A = -32 - M_B$$

$$M_A = -16 - M_B/2 \Rightarrow -16 - 0.5\,M_B$$

From (2) Equation

$$-16 - 0.5\,M_B + 8M_B + 3M_C = -392$$

$$\cancel{M_B} + 7.5\,M_B + 3M_C = -376 \rightarrow (4)$$

Sloving (3) & (4)

(3) Eq → Multiply 7.5

$$7.5\,\cancel{M_B} + 15\,M_C = -900$$

$$(4) \Rightarrow \underset{(-)}{}\ 7.5\,M_B + 3M_C = -376$$

$$12\,M_C = -524$$

$$\boxed{M_C = -43.667 \text{ KN-m}}$$

Substite in (3) eq

$$M_B + 2(-43.667) = -120$$

$$\boxed{M_B = -32.66 \text{ KN-m}}$$

UNIT- IV

Slope - Deflection method

Formulae :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

Directions:-

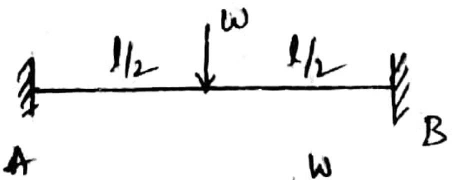
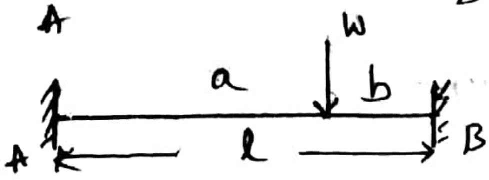
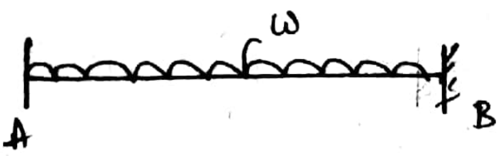
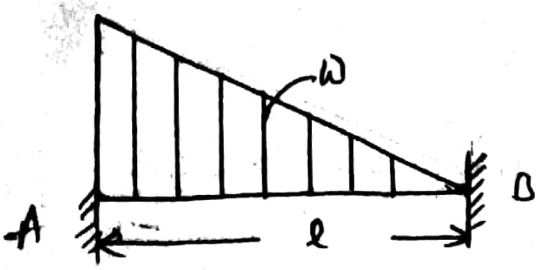
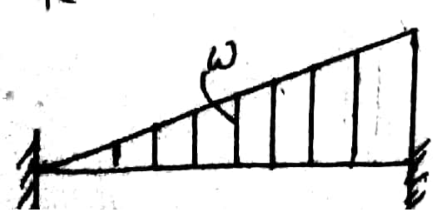
clockwise → positive

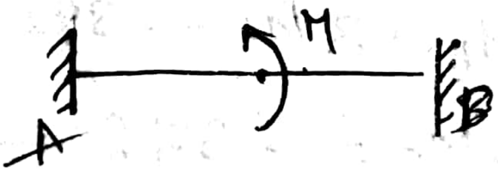
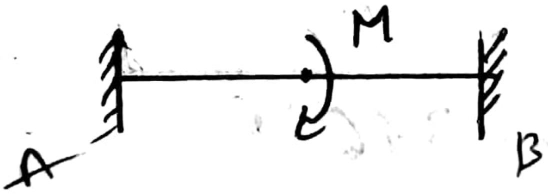
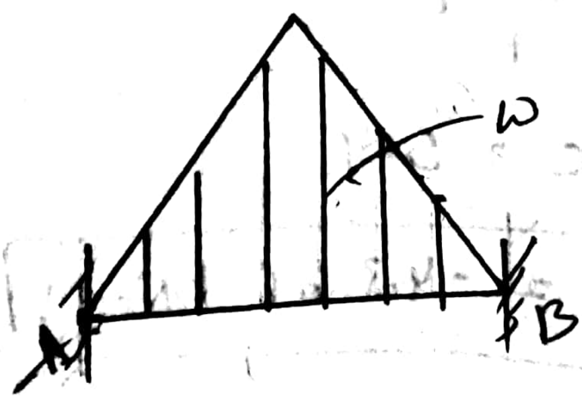
Anticlockwise → Negative

Note :-

→ clockwise moments are taken as postive.

→ Anticlockwise moments are taken as negative.

Loading diagram	M_{FAB}	M_{FBA}
	$-\frac{wl}{8}$	$\frac{wl}{8}$
	$-\frac{wab^2}{l^2}$	$\frac{wa^2b}{l^2}$
	$-\frac{wl^2}{12}$	$\frac{wl^2}{12}$
	$-\frac{wl^2}{20}$	$\frac{wl^2}{30}$
	$-\frac{wl^2}{30}$	$\frac{wl^2}{20}$

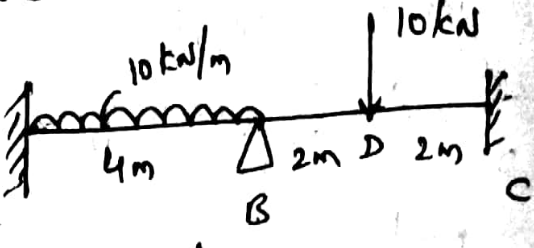
	M_{FAB}	M_{FBA}
	$-M/4$	$-M/4$
	$\frac{M}{4}$	$\frac{M}{4}$
	$-\frac{5wl^2}{96}$	$\frac{5wl^2}{96}$

Formulae :-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

Analyse the Continuous beam as shown in fig^n.



Step-1

Calculation of fixed end moments :-

$$M_{FAB} = -\frac{wl^2}{12}$$

$$= -\frac{10 \times 4^2}{12}$$

$$= -13.33 \text{ kN-m}$$

$$M_{FBA} = \frac{wl^2}{12}$$

$$= 13.33 \text{ kN-m}$$

$$M_{FBC} = -\frac{wl}{8}$$

$$= -\frac{10 \times 4}{8}$$

$$= -5 \text{ kN-m}$$

$$M_{FCB} = \frac{wl}{8}$$

$$= \frac{10 \times 4}{8}$$

$$= 5 \text{ kN-m}$$

Step-2:-

slope-deflection equations

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -13.33 + \frac{2EI}{4}\left[0 + \theta_B - \frac{3}{4}(0)\right]$$

$$M_{AB} = -13.33 + 0.5EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 13.33 + \frac{2EI}{4}\left(2\theta_B + 0 - \frac{3}{4}(0)\right]$$

$$M_{BA} = 13.33 + EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -5 + \frac{2EI}{4}\left(2\theta_B + 0 - \frac{3}{4}(0)\right]$$

$$M_{BC} = -5 + EI\theta_B$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 5 + \frac{2EI}{4}\left(0 + \theta_B - 0\right)$$

$$M_{CB} = 5 + 0.5EI\theta_B$$

Step-3:-

Equilibrium equations:-

(4) At any joint sum of moments equal to "zero"

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$13.33 + EI\theta_B - 5 + EI\theta_B = 0$$

$$2EI\theta_B + 8.33 = 0$$

$$2EI\theta_B = -8.33$$

$$EI\theta_B = -4.165$$

$$\boxed{EI\theta_B = -4.165}$$

Step-4:-

Final moments:-

$$M_{AB} = -13.33 + 0.5\,EI\theta_B$$

$$= -13.33 + 0.5(-4.165)$$

$$\boxed{M_{AB} = -15.4125 \text{ kN-m}}$$

$$M_{BA} = 13.33 + EI\theta_B$$

$$= 13.33 - 4.165$$

$$\boxed{M_{BA} = 9.165 \text{ kN-m}}$$

$$M_{BC} = -5 + EI\theta_B$$

$$= -5 - 4.165$$

$$\boxed{M_{BC} = -9.165 \text{ kN-m}}$$

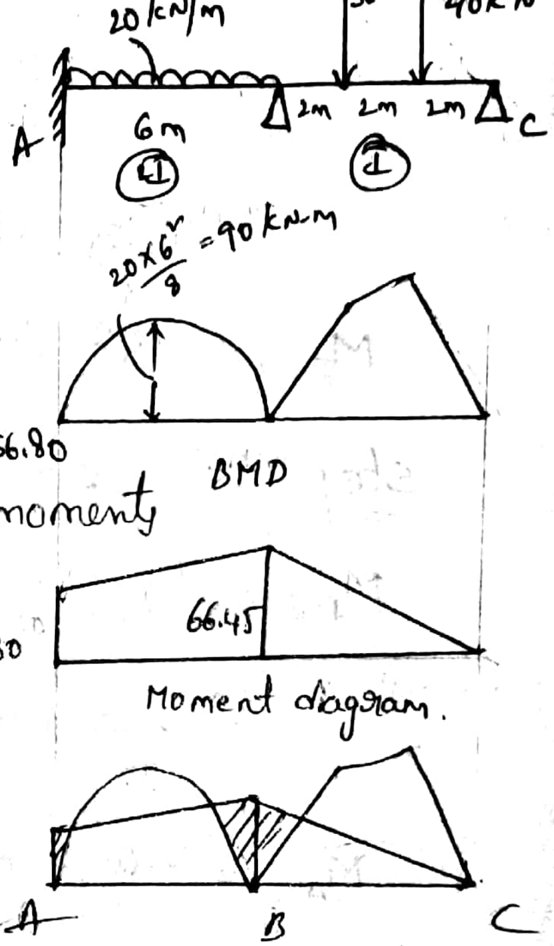
$M_{CB} = 5 + 0.5 EI\theta_B$

$= 5 + 0.5(-4.165)$

$= 2.917$ kN-m

$\boxed{M_{CB} = 2.917 \text{ kN-m}}$

17. Analyse the Continuouse beam as shown in fig?



Let M_A, M_B & M_C be the support moments and R_A, R_B & R_C be the support Reactions of the given beam.

Step 1:-

calculation of fixed end moments

$M_{FAB} = \frac{-wl^2}{12}$

$= -\frac{20 \times 6^2}{12}$

$M_{FAB} = -60$ kN-m

$M_{FBA} = \frac{wl^2}{12}$

$= \frac{20 \times 6^2}{12}$

$M_{FBA} = 60$ kN-m

$M_{FBC} = -\frac{w_1 a_1 b_1^2}{l^2} - \frac{w_2 a_2 b_2^2}{l^2}$

$= -\frac{30 \times 2 \times 4^2}{6^2} - \frac{40 \times 4 \times 2^2}{6^2}$

$$M_{FBC} = -44.44$$

$$M_{FCB} = +\frac{w_1 a_1^2 b_1}{l^2} + \frac{w_2 a_2^2 b_2}{l^2}$$

$$= \frac{30 \times 2^2 \times 4}{36} + \frac{40 \times 4^2 \times 2}{36}$$

$$= 13.33 + 35.55$$

$$= 48.88 \text{ kN-m}$$

$$M_{FCB} = \underline{48.88} \text{ kN-m}$$

Step 2 :- slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -60 + \frac{2E \times 2I}{6}\left[0 + \theta_B - \frac{3}{l}(0)\right]$$

$$M_{AB} = -60 + 0.66\,\theta_B EI$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 60 + \frac{2E \times 2I}{6_3}\left(2\theta_B + 0 - 0\right)$$

$$M_{BA} = 60 + 1.33\,\theta_B EI \quad (\text{or}) \quad 60 + \frac{4EI\theta_B}{3}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -44.44 + \frac{\cancel{2}EI}{\cancel{6}_3}\left(2\theta_B + \theta_C - 0\right)$$

$$M_{BC} = -44.44 + \frac{1}{3}EI\left(2\theta_B + \theta_C\right)$$

$$= -44.44 + \frac{2EI\theta_B}{3} + \frac{EI\theta_C}{3}$$

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$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_c + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 48.88 + \frac{2EI}{6}\left(2\theta_c + \theta_B - 0\right)$$

$$M_{CB} = 48.88 + \frac{1}{3}EI\left(2\theta_c + \theta_B\right)$$

$$= 48.88 + \frac{2EI\theta_c}{3} + \frac{EI\theta_B}{3}$$

Step 3:-

Equailibgium equations:- $1.33 =$

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

$$60 + 1.33\,\theta_B EI - 4444 + \frac{2EI\theta_B}{3} + \frac{EI\theta_c}{3} = 0$$

$$60 + \frac{4EI\theta_B}{3} - 44.44 + \frac{2EI\theta_B}{3} + \frac{EI\theta_c}{3} = 0$$

$$\frac{1}{3}\left(4EI\theta_B + 2EI\theta_B + EI\theta_c\right) = 44.44 - 60$$

$$= -15.56$$

$$4EI\theta_B + 2EI\theta_B + EI\theta_c = -46.68$$

$$6EI\theta_B + EI\theta_c = -46.68 \quad \text{--- (1)}$$

At 'c' $M_{CB} = 0$ (simply supported)

$M_{CB} = 0$

$$48.88 + \frac{2}{3}EI\theta_c + \frac{EI\theta_B}{3} = 0$$

$$\frac{1}{3}\left(2EI\theta_c + EI\theta_B\right) = -48.88$$

$$EI\theta_B + 2EI\theta_c = -146.64 \quad \text{--- (2)}$$

slove eq^n (1) & (2)

$$6EI\theta_B + EI\theta_C = -46.68$$

$$EI\theta_B + 2EI\theta_C = -146.64$$

② × 6 $\cancel{6EI\theta_B} + 12EI\theta_C = -879.84$

(1) × 1 $\cancel{6EI\theta_B} + EI\theta_C = -46.68$

$- \quad - \quad +$

$$11EI\theta_C = -833.16$$

$$\boxed{EI\theta_C = -75.74}$$

~~$6EI\theta_B$~~

$$EI\theta_B + 2EI\theta_C = -146.64$$

$$EI\theta_B + 2(-75.74) = -146.64$$

$$\boxed{EI\theta_B = 4.84}$$

Step 4 :-

(1) Final moments :-

$$M_{AB} = -60 + 0.66EI\theta_B$$

$$= -60 + 0.66(4.84)$$

$$\boxed{M_{AB} = -56.80 \text{ kN-m}}$$

$$M_{BA} = 60 + \frac{4}{3}EI\theta_B$$

$$= 60 + \frac{4}{3}(4.84)$$

$$\boxed{M_{BA} = 66.45 \text{ kN-m}}$$

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(26)

$$M_{BC} = -44.44 + \frac{2}{3}EI\theta_B + \frac{EI\theta_C}{3}$$

$$= -44.44 + \frac{2}{3} \times 4.84 + \frac{-75.74}{3}$$

$$\boxed{M_{BC} = -66.46 \text{ kN-m}}$$

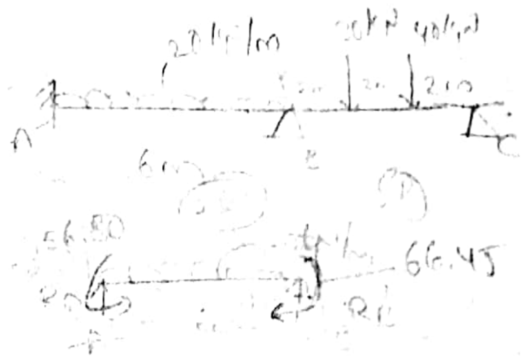
$$M_{CB} = 48.88 + \frac{2}{3}EI\theta_C + \frac{EI\theta_B}{3}$$

$$= 48.88 + \frac{2}{3} \times \overset{-75.74}{\cancel{4.84}} + \frac{4.84}{3}$$

$$\boxed{M_{CB} = 0 \text{ kN-m}}$$

Step 5:-

Reaction:-



Beam AB:-

$$R_A + R_B' = (20 \times 6)$$

$$\Sigma M_A = 0$$

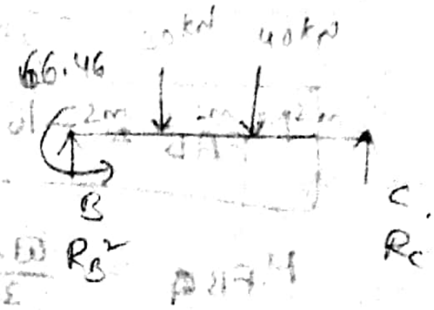
$$-R_B' \times 6 - 56.78 + 66.45 + 20 \times 6 \times \frac{6}{2} = 0$$

$$\boxed{R_B' = 61.61 \text{ kN}}$$

$$R_A = 120 - 61.61$$

$$= 58.39 \text{ kN}$$

$$\boxed{R_A = 58.39 \text{ kN}}$$



Beam BC:-

$$\Sigma V = 0$$

$$R_B^2 + R_c = 30 + 40$$

(27) $R_B^2 + R_C = 70\,kN$

$\Sigma M_A = 0$

$-R_C \times 6 + (40 \times 4) + (30 \times 2) - 66.46 = 0$

$$\boxed{R_C = 25.59\,kN}$$

$R_B^2 + R_C = 70$

$R_B^2 = 70 - 25.59$

$= 44.41\,kN$

$$\boxed{R_B^2 = 44.41\,kN}$$

$R_B = R_B^1 + R_B^2$

$= 61.61 + 44.41$

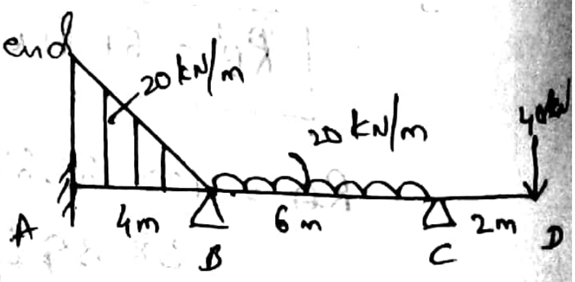
$$\boxed{R_B = 106.02\,kN}$$

Analyse the Continuous beam as shown in fig^n.

Sol:

step 1:-

calculation of fixed end moments



$M_{FAB} = \frac{-wl^2}{20}$

$= \frac{-20 \times 4^2}{20}$

$$\boxed{M_{FAB} = -16\,kN\text{-}m}$$

$M_{FBA} = \frac{wl^2}{30}$

$= \frac{20 \times 16}{30} \Rightarrow \boxed{M_{FBC} = 10.66\,kN\text{-}m}$

$$\cancel{M_{FBC} = -\frac{wl^2}{20}}$$

$$= \frac{-20 \times 6^2}{20}$$

$$\boxed{\cancel{M_{FBC} = -36 \text{ kN-m}}}$$

$$M_{FCB} = \frac{wl^2}{20}$$

$$\boxed{M_{FCB} = 36 \text{ kN-m}}$$

$$M_{FBC} = -\frac{wl^2}{12} = \frac{-20 \times 6^2}{12}$$

$$\boxed{M_{FBC} = -60 \text{ kN-m}}, \quad M_{FCB} = 60 \text{ kN-m}$$

$$M_{FCB} = 60 \text{ kN-m}$$

$$M_{CD} = -40 \times 2$$

$$= -80 \text{ kN-m}$$

$$\boxed{M_{CD} = -80 \text{ kN-m}}$$

step 2:-

slope deflection equations:-

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -16 + \frac{2EI}{4}(0 + \theta_B - 0)$$

$$M_{AB} = -16 + \frac{1}{2} EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 10.66 + \frac{2EI}{4}(2\theta_B + 0 - 0)$$

$$M_{BA} = 10.66 + EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -60 + \frac{\cancel{2}EI}{\cancel{6}_3}\left(2\theta_B + \theta_C\right)$$

$$M_{BC} = -60 + \frac{2EI\theta_B}{3} + \frac{EI\theta_C}{3}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 60 + \frac{\cancel{2}EI}{\cancel{6}_3}\left(2\theta_C + \theta_B - 0\right)$$

$$M_{CB} = 60 + \frac{2EI\theta_C}{3} + \frac{EI\theta_B}{3}$$

Step-3 :-

equilibrium equations.

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$10.66 + EI\theta_B - 60 + \frac{2EI\theta_B}{3} + \frac{EI\theta_C}{3} = 0$$

$$\therefore \left(\frac{2EI\theta_B}{} + EI\theta_C\right) + EI\theta_B = 60 - 10.66$$

$$10.66 + EI\theta_B - 60 + \frac{2EI\theta_B}{3} + \frac{EI\theta_C}{3} = 0$$

$$\cancel{10.66 + EI\theta_B - 60 + 0.66 EI\theta_B + 0.33 EI\theta_C = 0}$$

$$\cancel{1.66 EI\theta_B + 0.33 EI\theta_C} = 60 - 10.66$$

$$\frac{3EI\theta_B + 2EI\theta_B}{3} + \frac{EI\theta_C}{3} = 60 - 10.66$$

$$\frac{5EI\theta_B}{3} + \frac{EI\theta_C}{3} = 49.3$$

$$5EI\theta_B + EI\theta_C = 147.9 \quad \text{--- (1)}$$

$$M_{CB} + M_{CD} = 0$$

$$60 + \frac{2EI\theta_C}{3} + \frac{EI\theta_B}{3} - 80 = 0$$

$$\frac{1}{3}\left[2EI\theta_C + EI\theta_B\right] = 80 - 60$$

$$2EI\theta_C + EI\theta_B = 60 \quad \text{--- (2)}$$

$(2) \times 5 \rightarrow \quad \cancel{5EI\theta_B} + 10 EI\theta_C = 300$

$(1) \times 1 \rightarrow \quad \cancel{5EI\theta_B} + EI\theta_C = 147.9$

$\quad (-) \quad (-) \quad (-)$

$$9EI\theta_C = 152.1$$

$$\boxed{EI\theta_C = 16.9}$$

$$EI\theta_B + 2EI\theta_C = 60$$

$$\boxed{EI\theta_B = 26.22}$$

Step 4 :-

Final moments

$$M_{AB} = -16 + \frac{EI\theta_B}{2}$$

$$= -16 + \frac{26.22}{2}$$

$$\boxed{M_{AB} = -2.89 \text{ kN-m}}$$

$$M_{BA} = 10.66 + EI\theta_B$$

$$= 10.66 + 26.22$$

$$\boxed{M_{BA} = 36.88 \text{ kN-m}}$$

$$M_{BC} = -60 + \frac{2}{3}EI\theta_B + \frac{EI\theta_C}{3}$$

$$= -60 + \frac{2}{3}(26.22) + \frac{16.9}{3}$$

$$= -60 + 17.48 + 5.633$$

$$\boxed{M_{BC} = -36.88 \text{ kN-m}}$$

$$M_{CB} = 60 + \frac{2EI\theta_C}{3} + \frac{EI\theta_B}{3}$$

$$= 60 + \frac{2}{3} \times 16.9 + \frac{26.22}{3}$$

$$= 60 + 11.266 + 8.74$$

$$\boxed{M_{CB} = 80 \text{ kN-m}}$$

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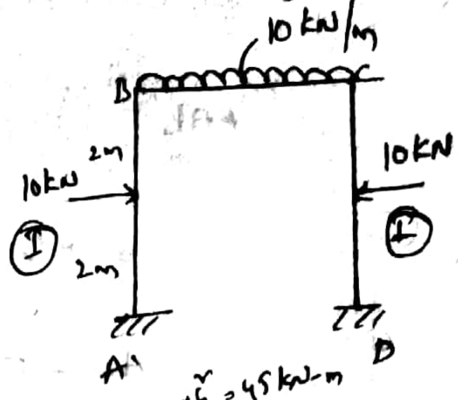
Analyse the portal frame as shown in fig^n.



Step 1:-

Calculation of fixed end moments

$M_{FAB} = \frac{-wl}{8}$

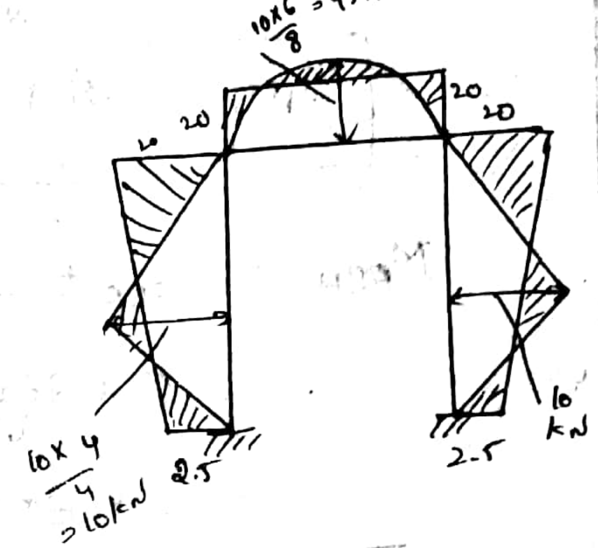
$= -\frac{10 \times 4}{8}$

$\boxed{M_{FAB} = -5 \text{ kN-m}}$

$M_{FBA} = \frac{wl}{8}$

$= \frac{10 \times 4}{8}$

$\boxed{M_{FBA} = 5 \text{ kN-m}}$



$M_{FBC} = \frac{-wl^2}{12}$

$= -\frac{10 \times 6^2}{12}$

$= -30$

$\boxed{M_{FBC} = -30 \text{ kN-m}}$

$M_{FCB} = \frac{wl^2}{12}$

$= \frac{10 \times 6^2}{12}$

$\boxed{M_{FCB} = 30 \text{ kN-m}}$

$M_{FCD} = -\frac{wl}{8} = -5 \text{ kN-m}$

$M_{FDC} = \frac{wl}{8} = 5 \text{ kN-m}$

step 2:-

slope deflection equations

$$M_{FAB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -5 + \frac{2EI}{\cancel{4}_2}\left(0 + \theta_B - 0\right)$$

$$= -5 + \frac{1}{2} \times EI\theta_B$$

$$\boxed{M_{AB} = -5 + \frac{EI\theta_B}{2}}$$

$$M_{FBA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 5 + \frac{2EI}{4}\left(2\theta_B + 0 - 0\right)$$

$$= 5 + EI\theta_B$$

$$\boxed{M_{FBA} = 5 + EI\theta_B}$$

$$M_{FBC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$= -30 + \frac{2E \times 2I}{6}\left(2\theta_B + \theta_C\right)$$

$$= -30 + \frac{2}{3}EI\left(2\theta_B + \theta_C\right)$$

$$\boxed{M_{BC} = -30 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 30 + \frac{2E \times 2I}{6}\left(2\theta_C + \theta_B\right)$$

$$\boxed{M_{CB} = 30 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B}$$

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$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\Delta}{l}\right)$$

$$= -5 + \frac{2EI}{4}(2\theta_C + 0 - 0)$$

$$= -5 + EI\theta_C$$

$$\boxed{M_{CD} = -5 + EI\theta_C}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\Delta}{l}\right)$$

$$= 5 + \frac{2EI}{4}(0 + \theta_C - 0)$$

$$= 5 + 0.5\,EI\theta_C$$

$$\boxed{M_{DC} = 5 + 0.5\,EI\theta_C}$$

Step 3:-

Equilibrium equations :-

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$5 + EI\theta_B - 30 + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 0$$

$$EI\theta_B + \frac{4}{3}EI\theta_B + \frac{2}{3}EI\theta_C = 30 - 5$$

$$\frac{3EI\theta_B + 4EI\theta_B + 2EI\theta_C}{3} = 25$$

$$7EI\theta_B + 2EI\theta_C = 75 \quad \text{— (1)}$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$30 + \frac{4}{3}EI\theta_C + \frac{2}{3}EI\theta_B - 5 + EI\theta_C = 0$$

$$\frac{4EI\theta_C}{3} + \frac{2}{3}EI\theta_B + EI\theta_C = -30 + 5$$

$$\frac{4EI\theta_C + 2EI\theta_B + 3EI\theta_C}{3} = -25$$

(31)

$2EI\theta_B + 7EI\theta_C = -75$ — ②

$7EI\theta_B + 2EI\theta_C = 75$ —(1)

$2EI\theta_B + 7EI\theta_C = -75$ —(2)

2×1 $\quad 14EI\theta_B + 4EI\theta_C = 150$

7×2 $\quad 14EI\theta_B + 49EI\theta_C = \cancel{-150}\ -525$

$(-) \quad (-) \quad (+)$

$\cancel{-45EI\theta_C = 800}$

$-45EI\theta_C = 675$

$\boxed{EI\theta_C = -15}$

$7EI\theta_B + 2EI\theta_C = 75$

$7EI\theta_B + 2(-15) = 75$

$7EI\theta_B = 75 + 30$

$\boxed{EI\theta_B = 15}$

step4:-

Final moments:

$M_{AB} = -5 + 0.5EI\theta_B$

$= -5 + 0.5(15)$

$\boxed{M_{AB} = 2.5\,kN\text{-}m}$

$M_{BA} = 5 + EI\theta_B$

$= 5 + 15$

$\boxed{M_{BA} = 20\,kN\text{-}m}$

$$M_{BC} = -30 + \frac{4}{3} EI\theta_B + \frac{2}{3} EI\theta_C$$

$$= -30 + \frac{4}{3}(15) + \frac{2}{3}(-15)$$

$$= -30 + 20 - 10$$

$$= -20$$

$$\boxed{M_{BC} = -20 \text{ kN-m}}$$

$$M_{CB} = 30 + \frac{4}{3} EI\theta_C + \frac{2}{3} EI\theta_B$$

$$= 30 + \frac{4}{3}(-15) + \frac{2}{3}(15)$$

$$= 30 - 20 + 10$$

$$\boxed{M_{CB} = 20 \text{ kN-m}}$$

$$M_{CD} = -5 + EI\theta_C$$

$$= -5 - 15$$

$$= -20 \text{ kN-m}$$

$$\boxed{M_{CD} = -20 \text{ kN-m}}$$

$$M_{DC} = 5 + 0.5 EI\theta_C$$

$$= 5 + 0.5(-15)$$

$$= 5 - 7.5$$

$$\boxed{M_{DC} = -2.5 \text{ kN-m}}$$

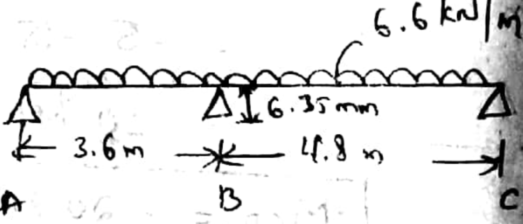
Analyse of Continuous beams with settlement of supports :-

Sign Convention:-

w.r.t left support, Right support is down then 'δ' is positive.

w.r.t left support, Right support is up then 'δ' is negative.

Analyse the continuous beam as shown in fig? If the support 'B' settles by 6.35 mm below the supports A & C. Take $E = 2\times10^5\ N/mm^2$ & $I = 3320\ cm^4$

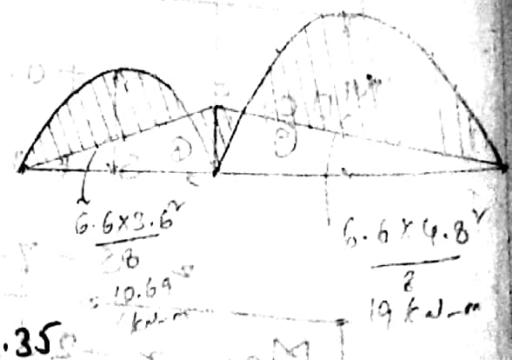


$E = 2\times10^5\ N/mm^2$

$I = 3320\ cm^4$

$= \frac{3320}{\cancel{10^{-4}}}$

$= 3320\times10^4\ mm^4$



$\delta = 6.35\ mm$

$EI\delta = 2\times10^5\times3320\times10^4\times6.35$

$= 4.21\times10^{13}\ N-mm^3$

$= \frac{4.21\times10^{13}}{10^3}\ kN$

$= \frac{4.21\times10^{10}}{(1000)^3} = 42.1\ kN-m^3$

(34) $\boxed{EI\delta = 42 \text{ kN-m}^2}$

step 1:-

Calculation of fixed end moments

$$M_{FAB} = -\frac{wl^2}{12}$$

$$= -\frac{6.6 \times (3.6)^2}{12}$$

$$= -7.128$$

$$\boxed{M_{FAB} = -7.128 \text{ kN-m}}$$

$$M_{FBA} = \frac{wl^2}{12}$$

$$= \frac{6.6 \times 3.6^2}{12}$$

$$\boxed{M_{FBA} = 7.128 \text{ kN-m}}$$

$$M_{FBC} = -\frac{wl^2}{12}$$

$$= \frac{-6.6 (4.8)^2}{12}$$

$$\boxed{M_{FBC} = -12.67 \text{ kN-m}}$$

$$M_{FCB} = \frac{wl^2}{12}$$

$$M_{FCB} = \frac{6.6 (4.8)^2}{12}$$

$$\boxed{M_{FCB} = 12.67 \text{ kN-m}}$$

(35) Step 2:-

Calculation of slope deflection eqⁿ.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -7.128 + \frac{2EI}{3.6}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -7.128 + \frac{4EI\theta_A}{3.6} + \frac{2EI\theta_B}{3.6} - \frac{6EI\delta}{3.6\times3.6}$$

$$= -7.128 + \frac{4EI\theta_A}{3.6} + \frac{2EI\theta_B}{3.6} - \frac{6(42.16)}{3.6^2}$$

$$M_{AB} = -26.64 + 1.1EI\theta_A + 0.55EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 7.128 + \frac{2EI}{3.6}\left(2\theta_B + \theta_A - \frac{3\times\delta}{3.6}\right)$$

$$= 7.128 + \frac{4EI\theta_B}{3.6} + \frac{2EI\theta_A}{3.6} - \frac{6EI\delta}{3.6^2}$$

$$= 7.128 + 1.1EI\theta_B + 0.55EI\theta_A - \frac{6\times(42.16)}{3.6^2}$$

$$M_{BA} = -12.39 + 1.1EI\theta_B + 0.55EI\theta_A$$

$$M_{BA} = -12.39 + 1.1EI\theta_B + 0.55EI\theta_A$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -12.67 + \frac{2EI}{4.8}\left(2\theta_B + \theta_C - \frac{3(\delta)}{4.8}\right)$$

$$= -12.67 + \frac{4EI}{4.8}\theta_B + \frac{2EI\theta_C}{4.8} + \frac{6EI\delta}{4.8^2}$$

$$= -12.67 + 0.83EI\theta_B + 0.416EI\theta_C + 6\times4$$

$M_{BC} = -1.69 + 0.83 EI\theta_B + 0.416 EI\theta_C$

$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$

$= 12.67 + \frac{2EI}{4.8}\left(2\theta_C + \theta_B - \frac{3(-\delta)}{4.8}\right)$

$= 12.67 + \frac{4EI\theta_C}{4.8} + \frac{2EI\theta_B}{4.8} + \frac{(42.16)3}{4.8^2}$

$= 23.65 + 0.83 EI\theta_C + 0.416 EI\theta_B$

$M_{CB} = 23.65 + 0.83 EI\theta_C + 0.416 EI\theta_B$

step 5:
Equilibrium equation

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

At joint B $\quad M_{BA} + M_{BC} = 0$

$-12.39 + 1.1 EI\theta_B + 0.55 EI\theta_A$ ~~$-12.67 + 0.83$~~ $-1.69 + 0.83 EI\theta_B + 0.416 EI\theta_C = 0$
~~$EI\theta_B + 0.416$~~

$-14.08 + 1.93 EI\theta_B + 0.558 EI\theta_A + 0.416 EI\theta_C = 0$

$0.55 EI\theta_A + 1.93 EI\theta_B + 0.416 EI\theta_C = 14.08 \text{ — (1)}$

$M_{AB} = 0$ (simply supported)

$-26.69 + 1.1 EI\theta_A + 0.56 EI\theta_B = 0$

$1.1 EI\theta_A = -0.55 EI\theta_B + 26.69$

$EI\theta_A = -0.5 EI\theta_B + 24.21$

Scanned by CamScanner

At support 'c' $M_{CB} = 0$

$23.65 + 0.83\, EI\theta_C + 0.416\, EI\theta_B = 0$

$0.416\, EI\theta_B + 0.83\, EI\theta_C = -23.65$

Divide the above the eq by 0.416

$EI\theta_B + 2EI\theta_C = -56.85$ — (2)

$EI\theta_A = -0.5\, EI\theta_B + 24.21$

Substitute the value $EI\theta_A$ in eq (1)

$0.55\, EI\theta_A + 1.93\, EI\theta_B + 0.416\, EI\theta_C = 14.08$

$0.55(24.21 - 0.5\, EI\theta_B) + 1.93\, EI\theta_B + 0.416\, EI\theta_C = 14.08$

$13.31 - 0.275\, EI\theta_B + 1.93\, EI\theta_B + 0.416\, EI\theta_C = 14.08$

$1.65\, EI\theta_B + 0.416\, EI\theta_C = 0.77$ — (3)

(2) × 1.65 ⇒

$\cancel{1.65\, EI\theta_B} + 3.31\, EI\theta_C = -93.80$

(3) $\cancel{1.65\, EI\theta_B} + 0.416\, EI\theta_C = 0.77$

(−) (−) (−)

$2.894\, EI\theta_C = -93.03$

$\boxed{EI\theta_C = -32.14}$

$EI\theta_B + 2EI\theta_C = -56.85$

$EI\theta_B + 2(-32.14) = -56.85$

$\boxed{EI\theta_B = 7.43}$

$$EI\theta_A = -0.5 EI\theta_B + 24.21$$

$$= -0.5(7.43) + 24.21$$

$$\boxed{EI\theta_A = 20.49}$$

Step 4:-

Calculate the final moments.

$$M_{BA} = -12.39 + 1.1 EI\theta_B + 0.55 EI\theta_A$$

$$= -12.39 + 1.1(7.43) + 0.55(20.49)$$

$$\boxed{M_{BA} = 7.05 \text{ kN-m}}$$

$$M_{AB} = -26.64 + 1.1 EI\theta_A + 0.55 EI\theta_B$$

$$= -26.64 + 1.1(20.49) + 0.55(7.43)$$

$$\boxed{M_{AB} = 0}$$

$$M_{BC} = -1.69 + 0.83 EI\theta_B + 0.416 EI\theta_C$$

$$= -1.69 + 0.83(7.43) + 0.416(-32.14)$$

$$\boxed{M_{BC} = -\cancel{8.89}\ \text{kN-m}}$$

7.05

$$M_{CB} = 0$$

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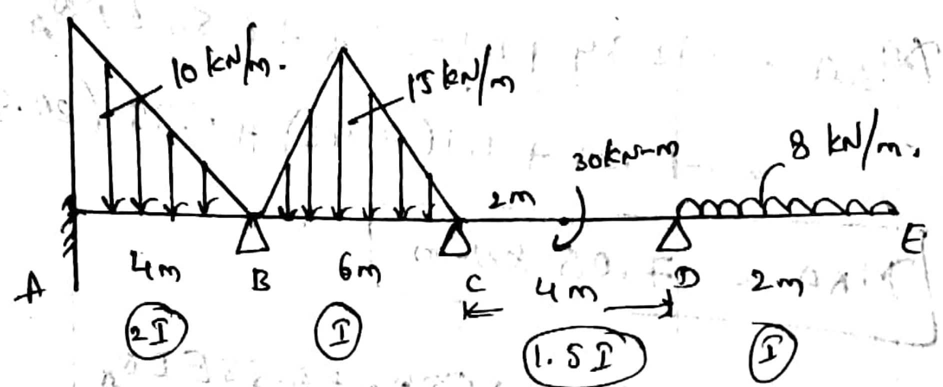
Analyse the Continuouse beam as shown in fig.



Step 1:-

calculation of fixed end moment

$$M_{FAB} = \frac{-wl^2}{20}$$

$$= \frac{-10 \times 4^2}{20}$$

$$= -8 \text{ kN-m}$$

$$\boxed{M_{FAB} = -8 \text{ kN-m}}$$

$$M_{FBA} = \frac{wl^2}{20}$$

$$= \frac{10 \times 4^2}{30}$$

$$\boxed{M_{FBA} = 5.33 \text{ kN-m}}$$

$$M_{FBC} = -\frac{5wl^2}{96}$$

$$= -\frac{5 \times 15 \times 6^2}{96}$$

$$= -28.125 \text{ kN-m}$$

$$\boxed{M_{FBC} = -28.12 \text{ kN-m}}$$

$$M_{FCB} = \frac{5wl^2}{96}$$

$$= \frac{5 \times 15 \times 6^2}{96}$$

$$\boxed{M_{FCB} = 28.12 \text{ kN-m}}$$

$$M_{FCD} = M/4$$

$$= \frac{30}{4}$$

$$= 7.5 \text{ kN-m}$$

$$\boxed{M_{FCD} = M_{FDC} = 7.5 \text{ kN-m}}$$

$$M_{DE} = -8 \times 2 \times 2/2$$

$$= -16 \text{ kN-m}$$

$$\boxed{M_{DE} = -16 \text{ kN-m}}$$

step 2:-

Calculation of slope deflection eq^n.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -8 + \frac{2E \times 2I}{4}(0 + \theta_B - 0)$$

$$= -8 + EI\theta_B$$

$$\boxed{M_{AB} = -8 + EI\theta_B}$$

$$= 5.33 + \frac{2}{3} EI\theta_B$$

$$\cancel{M_{BA} = 5.33 + \frac{2}{3} EI\theta_B}$$

$$\cancel{M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)}$$

$$\cancel{= -28.12 + \frac{2E \times 1.5I}{4}}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 5.33 + \frac{2E \times 2I}{6}(2\theta_B)$$

$$= 5.33 + \frac{1}{3}EI\theta_B$$

$$\boxed{M_{BA} = 5.33 + 1.33 EI\theta_B}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -28.12 + \frac{2E \times I}{6}(2\theta_B + \theta_C - 0)$$

$$= -28.12 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_C$$

$$\boxed{M_{BC} = -28.12 + 0.66 EI\theta_B + 0.33 EI\theta_C}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 28.12 + \frac{2EI}{6}(2\theta_C + \theta_B - 0)$$

$$= 28.12 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B$$

$$M_{CB} = 28.12 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$$

$$= 7.5 + \frac{2E \times 1.5I}{4}\left(2\theta_C + \theta_D - 0\right)$$

$$= 7.5 + \frac{3}{4}EI\left(2\theta_C + \theta_D\right)$$

$$\boxed{M_{CD} = 7.5 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_D}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$$

$$= 7.5 + \frac{2E \times 1.5I}{4}\left(2\theta_D + \theta_C - 0\right)$$

$$= 7.5 + \frac{3}{4}EI\left(2\theta_D + \theta_C\right)$$

$$\boxed{M_{DC} = 7.5 + \frac{3}{2}EI\theta_D + \frac{3}{4}EI\theta_C}$$

Step 3 :-

Equilibrium equations

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$5.33 + 2EI\theta_B + 28.12 + 0.66EI\theta_B + 0.33EI\theta_C = 0$$

$$-22.79 + 2.66EI\theta_B + 0.33EI\theta_C = 0$$

$$2.66EI\theta_B + 0.33EI\theta_C = 22.79 \quad \text{— (1)}$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$28.12 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B + 7.5 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_D$$

$$35.62 + 0.66EI\theta_C + 0.33EI\theta_B + 1.5EI\theta_C + 0.75EI\theta_D$$

$$35.62 + 2.16EI\theta_C + 0.33EI\theta_B + 0.75EI\theta_D = 0$$

$$2.16EI\theta_C + 0.33EI\theta_B + 0.75EI\theta_D = -35.62 \quad \text{— (2)}$$

In ① eqⁿ

$$2.66 EI\theta_B + 0.33 EI\theta_C = 22.79$$

$$2.66 EI\theta_B = 22.79 - 0.33 EI\theta_C$$

$$EI\theta_B = 8.56 - 0.125 EI\theta_C$$

$$\boxed{EI\theta_B = 8.55 - 0.125 EI\theta_C}$$

02/17

from eqⁿ(2)

$$2.16 EI\theta_C + 0.33 EI\theta_B + 0.75 EI\theta_D = -35.62$$

$$2.16 EI\theta_C + 0.33(8.55 - 0.125 EI\theta_C) + 0.75 EI\theta_D = -35.62$$

$$2.16 EI\theta_C + 2.82 - 0.04 EI\theta_C + 0.75 EI\theta_D = -35.6$$

$$2.118 EI\theta_C + 0.75 EI\theta_D = -38.44 \quad \text{—(3)}$$

~~from eqⁿ(1)~~

$$\cancel{2.66 EI\theta_B + 0.33 EI\theta_C = 22.79}$$

$$\cancel{2.66(8.55 - 0.125 EI\theta_C) + 0.33 EI\theta_C = 22.79}$$

$$\cancel{22.743 - 0.3325 EI\theta_C + 0.33 EI\theta_C = 22.79}$$

$$\cancel{-2.5\times10^{-3} EI\theta_C = 0.047}$$

$$\cancel{EI\theta_C =}$$

$$M_{DC} + M_{DE} = 0$$

$$7.5 + 1.5 EI\theta_D + 0.75 EI\theta_C - 16 = 0$$

$$-8.5 + 1.5 EI\theta_D + 0.75 EI\theta_C = 0$$

$$1.5 EI\theta_D + 0.75 EI\theta_C = 8.5 \quad \text{—(2)}$$

$3\times2 \rightarrow \quad 4.237\,EI\theta_c + \cancel{1.5\,EI\theta_D} = -76.84$

$\quad\; \underline{\overset{(-)}{0.75\,EI\theta_c} \overset{(-)}{+} \cancel{1.5\,EI\theta_D} = \overset{(-)}{8.5}}$

$3.487\,EI\theta_c = -85.35$

$$\boxed{EI\theta_c = -24.47}$$

eqⁿ (2)

$1.5\,EI\theta_D + 0.75\,EI\theta_c = 8.5$

$1.5\,EI\theta_D + 0.75(-24.47) = 8.5$

$1.5\,EI\theta_D = 26.8$

$$\boxed{EI\theta_D = 17.86}$$

$EI\theta_B = 8.55 - 0.125\,EI\theta_c$

$= 8.55 - 0.125(-24.47)$

$= 11.6$

$$\boxed{EI\theta_B = 11.6}$$

Step 4 :-

Calculate the final moments :-

$M_{AB} = -8 + EI\theta_B$

$= -8 + 11.6$

$$\boxed{M_{AB} = 3.6\ kN\text{-}m}$$

$M_{BA} = 5.33 + 2EI\theta_B$

$= 5.33 + 2(11.6)$

$= 28.53$

$\boxed{M_{BA} = 28.53 \text{ kN-m}}$

$M_{BC} = -28.12 + 0.66EI\theta_B + 0.33EI\theta_C$

$= -28.12 + 0.66(11.6) + 0.33(-24.47)$

$= -28.53$ kN-m

$\boxed{M_{BC} = -28.53 \text{ kN-m}}$

$M_{CB} = 28.12 + \frac{2}{3}EI\theta_C + \frac{1}{3}EI\theta_B$

$= 28.12 + \frac{2}{3}(-24.47) + \frac{1}{3}(11.6)$

$= 15.67$ kN-m

$\boxed{M_{CB} = 15.67 \text{ kN-m}}$

$M_{CD} = 7.5 + \frac{3}{2}EI\theta_C + \frac{3}{4}EI\theta_D$

$= 7.5 + \frac{3}{2}(-24.47) + \frac{3}{4}(A.86)$

$\boxed{M_{CD} = -15.81 \text{ kN-m}}$

$$M_{DC} = 7.5 + \frac{3}{2} EI\theta_D + \frac{3}{4} EI\theta_C$$

$$= 7.5 + 1.5(17.86) + 0.75(-24.47)$$

$$= \underline{\underline{15.81}}$$

$$\boxed{M_{DC} = 15.81 \text{ KN-m}}$$

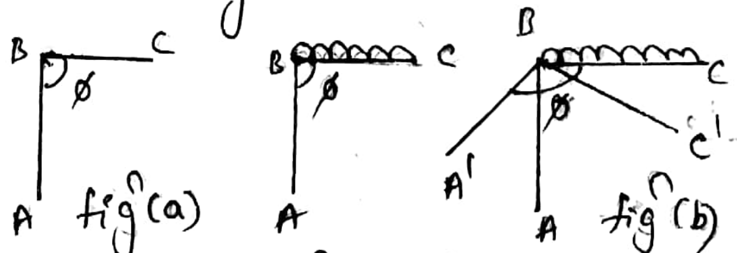
6/2/17 (39)

Degree of freedom:-

when a structure is loaded it deforms into a unique shape. The deformation of the structure can be completely specified provided the displacement of a no. of specified points on the structure. These displacements are refer to as the degree of freedom of the structure.

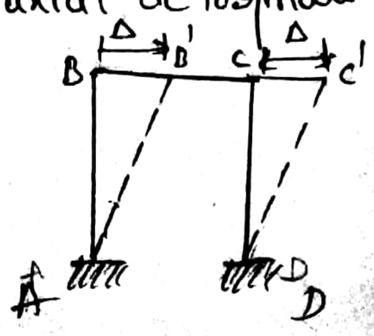
Assumptions of slope deflection equation:-

1) All the joints are rigid that is the angle b/w any two members in a joint doesn't changed even after deformation due to loading.



At joint 'B' in fig(a) the angle b/w any two members BC & BA remains 'θ'. even after deformation as shown in fig(b)

2) Displacement due to axial deformation are neglected.



3) shear deformations are neglected.

Sign Conventions:-

1. clockwise moments are positive, Anticlockwise moments are negative.

2. The clock wise rotation at the joints are postive, Anti clockwise rotation at the joints are negative.

3. settlement Δ is positive, if right side support is below the left side support.

4. settlement Δ is negative, if right side support is up the left side support.

UNIT - VI
-STRAIN ENERGY-

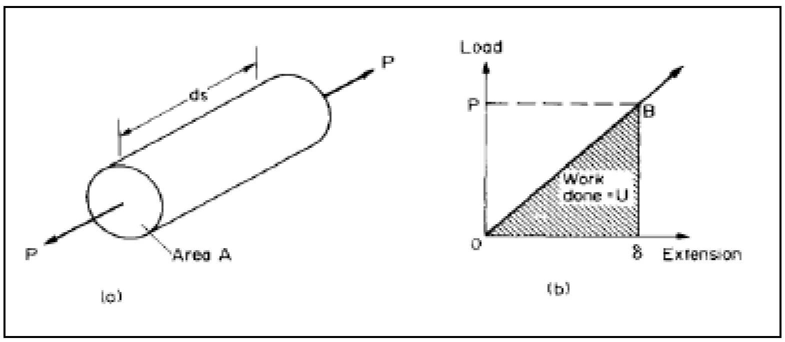
Introduction: -

Strain energy is as the energy which is stored within a material when work has been done on the material. Here it is assumed that the material remains elastic whilst work is done on it so that all the energy is recoverable and no permanent deformation occurs due to yielding of the material,

Strain energy U = work done

Thus for a gradually applied load the work done in straining the material will be given by the shaded area under the load-extension graph of Fig.

$$U = P\ \delta$$



Work done by a gradually applied load.

The unshaded area above the line OB of Fig. 7.1 is called the complementary energy, a quantity which is utilized in some advanced energy methods of solution and is not considered within the terms of reference of this text.

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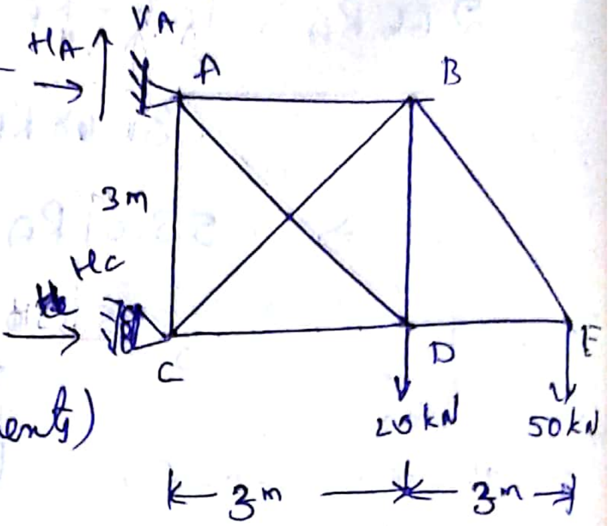
Pin - jointed frames:-

Find the force in the member BC in a pin jointed truss as shown in fig^n. Assume All the members have the same c/s area and modulus of elasticity.



Static indeterminacy:-

External indeterminacy

$D_e = r - 3$

(r = Reaction Components)

$= r - 3$

$= 3 - 3$

$D_e = \underline{0}$

Internal indeterminacy $D_i = m - (2j - r)$

$= 8 - (2 \times 5 - 3)$

$= 8 - 7$

$D_i = \underline{1}$

$\therefore$ Total static indeterminacy $= 0 + 1$

$= \underline{1}$

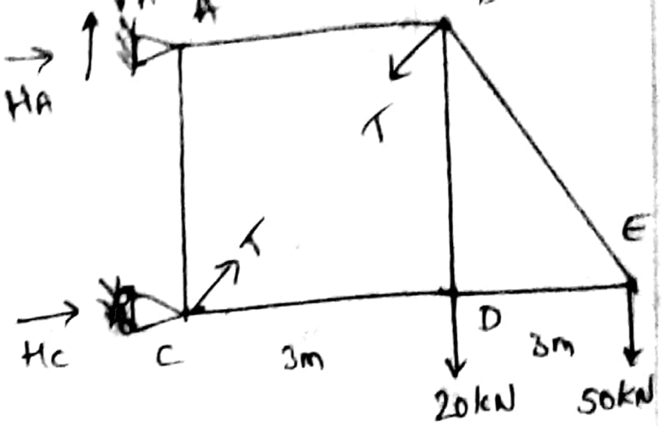
Let 'T' be the force in the member 'BC',

Now remove the member BC & apply a force 'T' in the member as shown in fig^n.

Reactions:-

$\Sigma H = 0$

$H_A + H_C = 0$



$\Sigma V = 0$

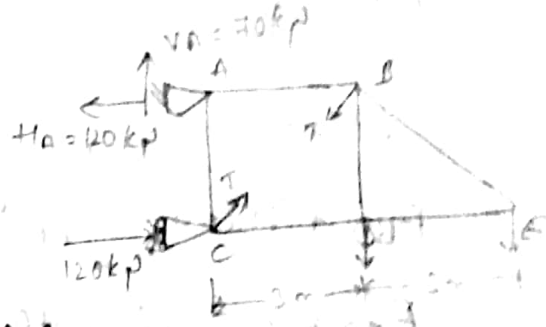
$V_A - 20 - 50 = 0$

$\boxed{V_A = 70 kN}$

$\Sigma M_A = 0$

$-H_C \times 3 + (20\times3) + (50\times6) = 0$

$\boxed{H_C = 120 kN}$



$H_A + H_C = 0$

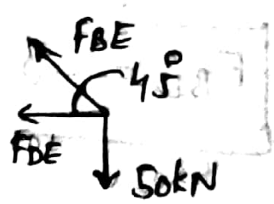
$H_A = -H_C$

$= -120 kN$

$H_A = 120 kN \ (\xleftarrow{H_A})$

Method of joints :-

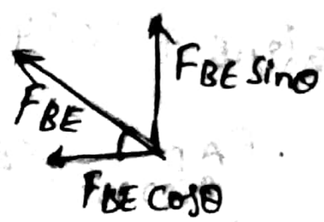
Joint E :-



$\tan\theta = \frac{3}{3}$

$\tan\theta = 1$

$\theta = 45°$

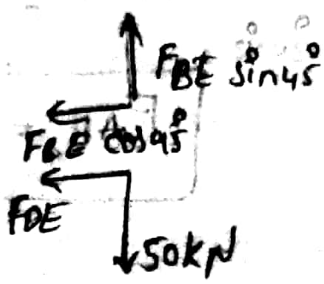


$\Sigma V = 0$

$F_{BE} \sin 45° = 50$

$F_{BE} = \frac{50}{\sin 45°}$

$\boxed{F_{BE} = 70.71}$ kN



$\Sigma H = 0$

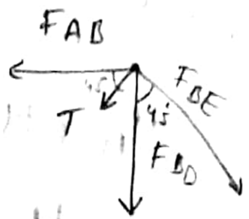
$F_{BE} \cos 45^\circ + F_{DE} = 0$

$(70.71) \cos 45^\circ + F_{DE} = 0$

$F_{DE} = -50\,kN$

$\boxed{F_{DE} = 50\,kN}$ (compression)

Joint B :-



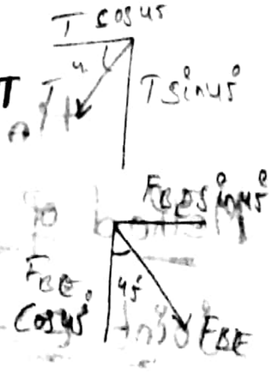
$\Sigma V = 0$

$F_{BD} + T\sin 45^\circ + F_{BE} \cos 45^\circ = 0$

$F_{BD} + T\sin 45^\circ + 70.71 \cos 45^\circ = 0$

$F_{BD} = -(70.71)\cos 45^\circ - 0.707T$

$= -0.707T - 50$



$\boxed{F_{BD} = -0.707T - 50}$

$\Sigma H = 0$

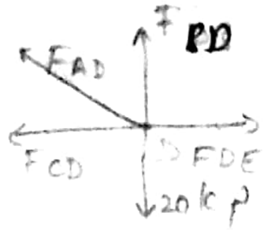
$F_{AB} + T\cos 45^\circ = F_{BE} \sin 45^\circ$

$F_{AB} = F_{BE} \sin 45^\circ - T\cos 45^\circ$

$= 50 - 0.707T$

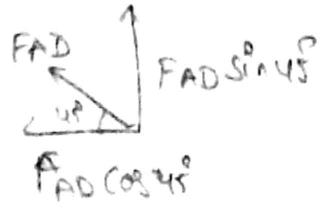
$\boxed{F_{AB} = 50 - 0.707T}$

Joint D:-



$\Sigma V = 0$

$F_{BD} + F_{AD} \sin 45^\circ = 20$



$\cancel{F_{DB}}$

$F_{AD} \sin 45^\circ = 20 - F_{BD}$

$F_{AD} \sin 45^\circ = 20 - (-0.707T - 50)$

$= 20 + 0.707T + 50$

$F_{AD} = 28.28 + T + 70.71$

$F_{AD} = 28.28 + T + 70.71$

$F_{AD} = 98.99 \simeq 99 + T$

$\boxed{F_{AD} = 99 + T}$

$\Sigma H = 0$

$F_{CD} + F_{AD} \cos 45^\circ = F_{DE}$

$F_{CD} = F_{DE} - F_{AD} \cos 45^\circ$

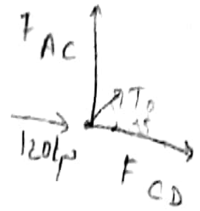
$= -50 - (99 + T) \cos 45^\circ$

$\Rightarrow -50 - 70 - 0.707T$

$\Rightarrow -120 - 0.707T$

$\boxed{F_{CD} = -120 - 0.707T}$

Joint C :-

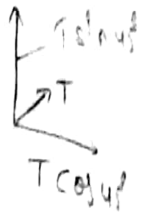


$\Sigma V = 0$

$F_{AC} + T\sin 45° = 0$

$F_{AC} = -0.707T$

$\boxed{F_{AC} = -0.707T}$



$\Sigma H = 0$

$120 + F_{CD} + T\cos 45° = 0$

$F_{CD} = -120 - T\cos 45°$

03/17

$\boxed{F_{CD} = -120 - 0.707T}$

$\sqrt{3^2+3^2} = 4.24$

Member	Force (F)	$\frac{\partial F}{\partial T}$	length (l)	$F \cdot \frac{\partial F}{\partial T} \cdot l$	Final force
AB	$-0.707T + 50$	-0.707	3m	$1.499T - 106.05$	82.92 kN ~~[illegible]~~ kN
AC	$-0.707T$	-0.707	3m	$1.499T$	32.92 kN ~~[illegible]~~ kN
AD	$99 + T$	1	4.24m	$419.76 + 4.24T$	52.43 kN ~~[illegible]~~ kN
BC	T	1	4.24m	$4.24T$	-46.57 kN ~~[illegible]~~
BD	$-0.707T - 50$	-0.707	3m	$1.499T + 106.05$	-17.07
BE	70.71	0	4.24m	0	70.71
CD	$-120 - 0.707T$	-0.707	3m	$254.52 + 1.499T$	-87.07
DE	-50	0	3m	0	-50
				$674.28 + 14.476T$	

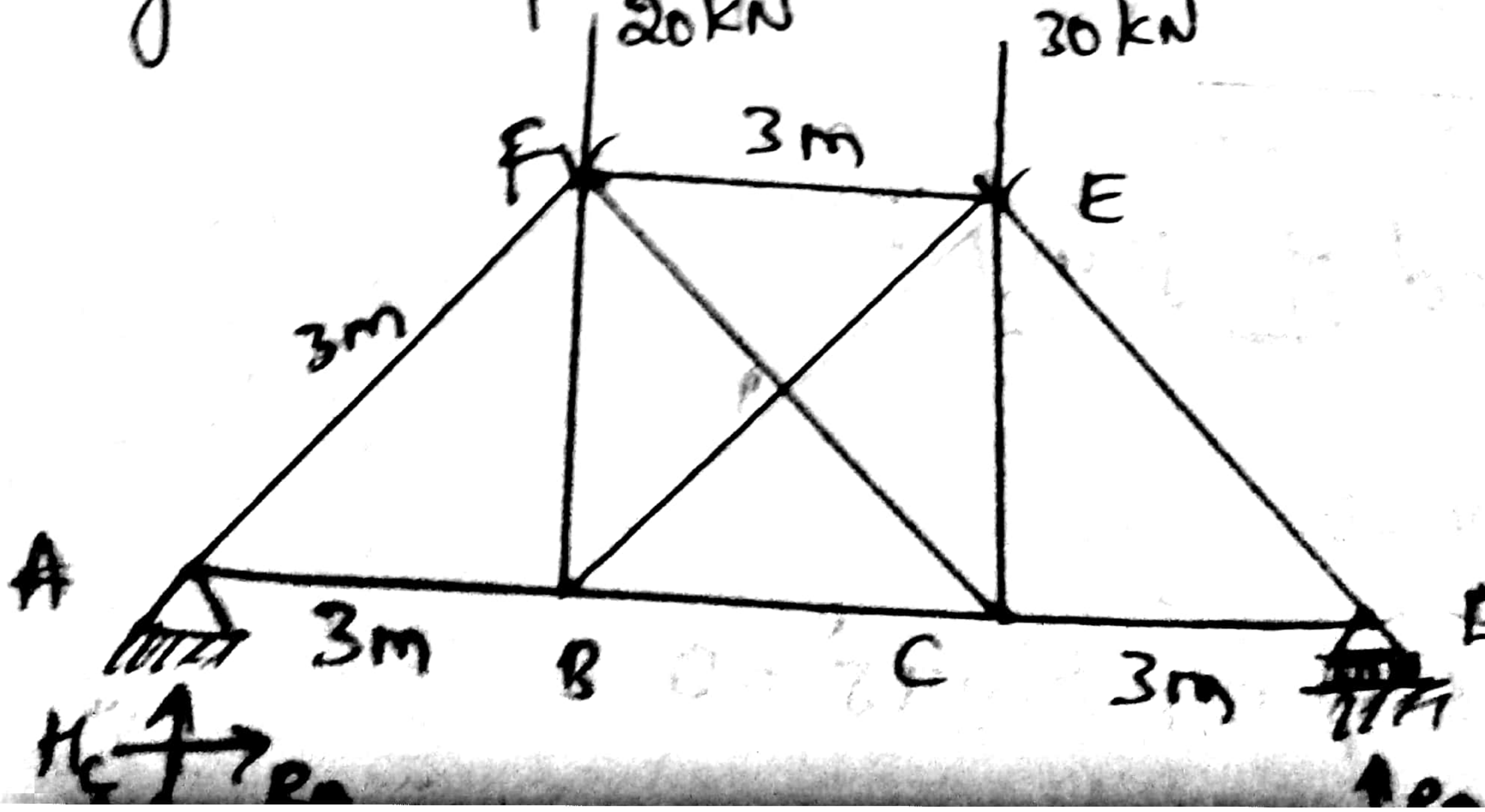
$$\Sigma F \cdot \frac{\partial F}{\partial T} \cdot l = 0$$

$$674.28 + 14.476T = 0$$

$$T = \frac{-674.28}{14.476}$$

$$\boxed{T = -46.57\ kN}$$

nalyse the frame



UNIT-VI
MOVING LOADS AND INFLUENCE LINES

Definitions of influence line

✸ An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
✸ An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point.
✸ An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

In engineering, an **influence line** graphs the variation of a function (such as the shear felt in a structure member) at a specific point on a beam or truss caused by a unit load placed at any point along the structure. Some of the common functions studied with influence lines include reactions (the forces that the structure's supports must apply in order for the structure to remain static), shear, moment, and deflection (Deformation). Influence lines are important in designing beams and trusses used in bridges, crane rails, conveyor belts, floor girders, and other structures where loads will move along their spanThe influence lines show where a load will create the maximum effect for any of the functions studied.

Influence lines are both scalar and additiveThis means that they can be used even when the load that will be applied is not a unit load or if there are multiple loads applied. To find the effect of any non-unit load on a structure, the ordinate results obtained by the influence line are multiplied by the magnitude of the actual load to be applied. The entire influence line can be scaled, or just the maximum and minimum effects experienced along the line. The scaled maximum and minimum are the critical magnitudes that must be designed for in the beam or truss

In cases where multiple loads may be in effect, the influence lines for the individual loads may be added together in order to obtain the total effect felt by the structure at a given point. When adding the influence lines together, it is necessary to include the appropriate offsets due to the spacing of loads across the structure. For example, a truck load is applied to the structure. Rear axle, B, is three feet behind front axle, A, then the effect of A at x feet along the structure must be added to the effect of B at $(x - 3)$ feet along the structure—not the effect of B at x feet along the structure.

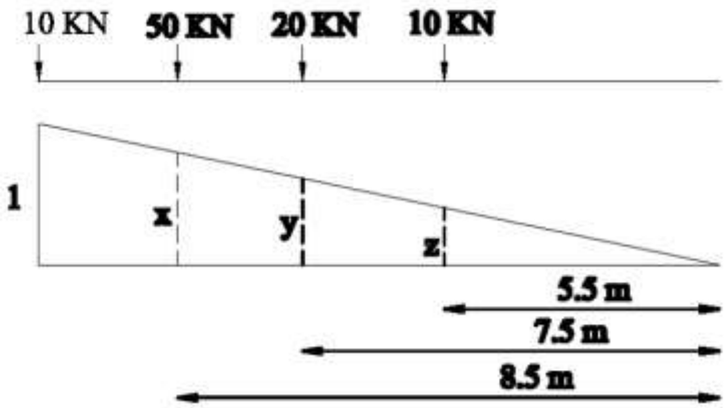
Many loads are distributed rather than concentrated. Influence lines can be used with either concentrated or distributed loadings. For a concentrated (or point) load, a unit point load is moved along the structure. For a distributed load of a given width, a unit-distributed load of the same width is moved along the structure, noting that as the load nears the ends and moves off the structure only part of the total load is carried by the structure. The effect of the distributed unit load can also be obtained by integrating the point load's influence line over the corresponding length of the structures.

1) *A system of concentrated load, role beam left to right, s.s beam span of 10m and 10 KN load leading*

Find 1.Absolute max +ve S.F

2. .Absolute max -ve S.F

3..Absolute max BM

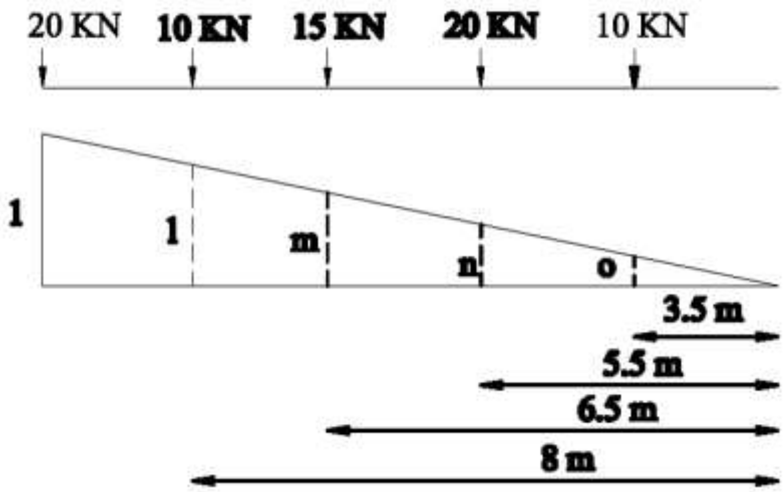


Solution

1. **Absolute max +ve S.F**

Using the similar triangle method and we get the x, y & z values

X	=	0.85 m
Y	=	0.75 m
Z	=	0.55 m
S.F	=	(10×1)+(15×0.83)+(20×0.75)+(10×0.55)
	=	43.25 KN

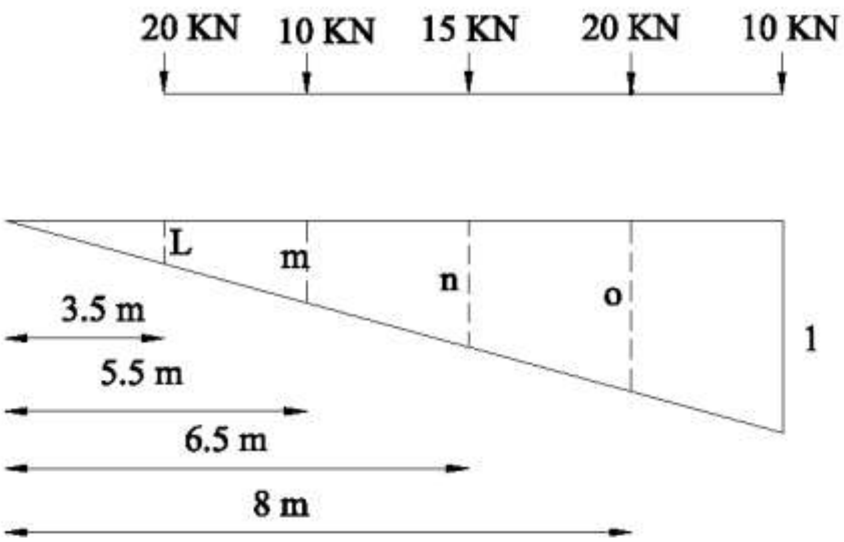


Using the similar triangle method and we get the l, m, n & o values

L=0.8 m

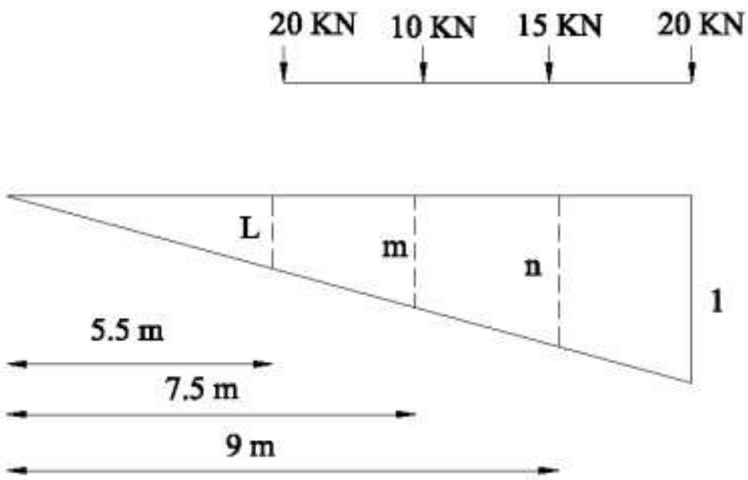
M	=	0.65 m
N	=	0.55 m
O	=	0.35 m
S.F	=	(20×1)+(10×0.8)+(15×0.65)+(20×0.55)+(10×0.35)
	=	52.25 KN

Absolute max -ve S.F



Using the similar triangle method and we get the l, m, n & o values

L	=	0.35 m
M	=	0.55 m
N	=	0.7 m
O	=	0.8 m
S.F	=	(10×1)+(20×0.8)+(15×0.7)+(10×0.55)+(20×0.35)
	=	- 49 KN



Using the similar triangle method and we get the l,m, & n values

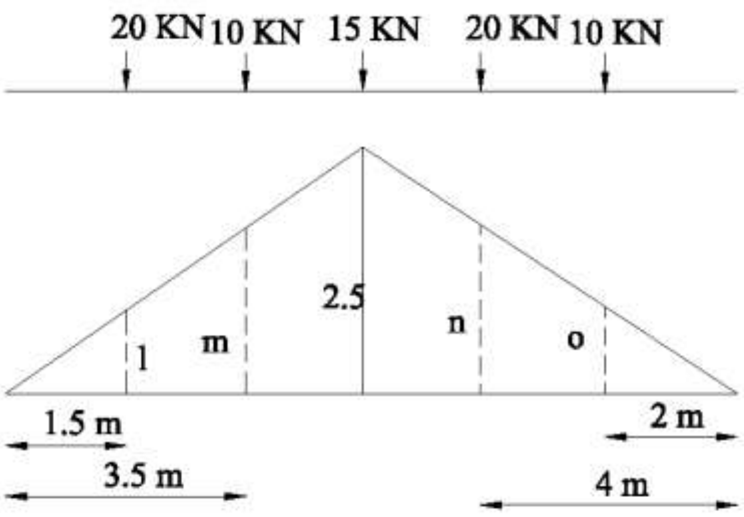
L=0.55 m

M=0.75 m

N=0.85 m

S.F=-((20×1)+(15×0.9)+(10v0.75)+(20×0.55))=-52 KN

(iii) Absolute max BM



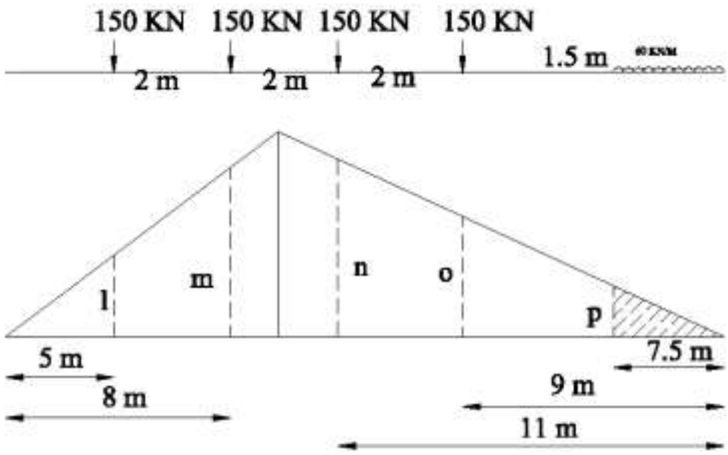
Using the similar triangle method and we get the l, m, n & o values

L	=	0.75 m
M	=	1.75 m
N	=	2 m
O	=	1 m
Max BM	=	(20×0.75) +(10×1.75)+(15×2.5)+(20×2)+(10×1)
	=	22.75 KN

2) ***<u>The four equal loads of 150 KN ,each equally spaced at apart 2m and UDL of 60 KN/m at a distance of 1.5m from the last 150 KN loads cross a girder of 20m from span R to L.Using influence line ,calculate the S.F and BM at a section of 8m from L.H.S support when leading of 150KN 5m from L.H.S.</u>***

<u>Solution</u>

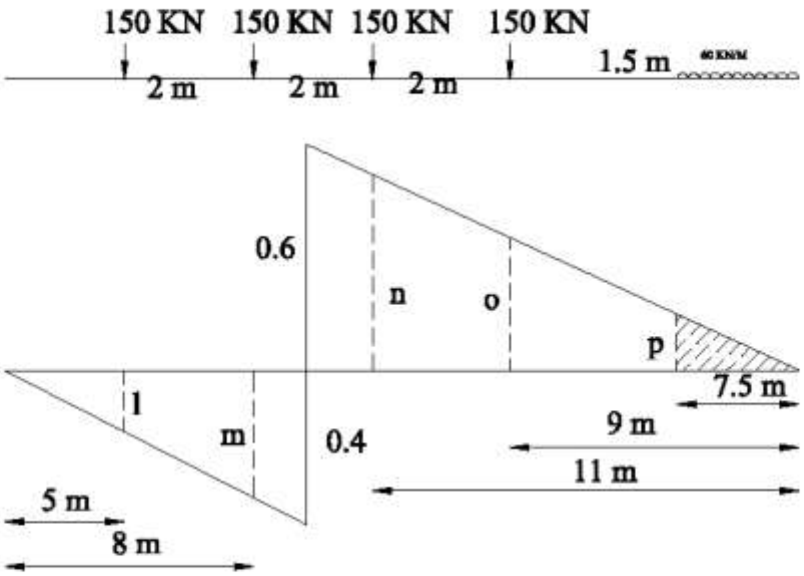
(i) Max BM



L	=	3 m
M	=	4.2 m
N	=	4.4 m
0	=	3.6 m
P	=	3 m
A	=	11.25 m^2
BM	=	(150×3)+(150×4.2)+(150×4.4)+(150×306)+(60×11.25)
	=	2955 KNm

ii) Shear Force

Compute maximum end shear for the given beam loaded with moving loads as shown in Figure



L = 0.25 m,

M = 0.3 m,

N = 0.55 m,

O = 0.45 m,

P = 0.375 m

SF = $((150\times0.25)+(150\times0.35)+(150\times0.55)+(150\times0.45)+(60\times1.41))$

= 144. KN

Where do you get rolling loads in practice?

- Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

Name the type of rolling loads for which the absolute maximum bending moment occurs at the midspan of a beam.

- Single concentrated load
- udl longer than the span
- udl shorter than the span
- Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

What is meant by absolute maximum bending moment in a beam?

- When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section.
- The maximum of these bending moments will usually occur near or at the midspan.
- The maximum of maximum bending moments is called the absolute maximum bending moment.

Where do you have the absolute maximum bending moment in a simply supported beam when a series of wheel loads cross it?

- When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load Wcr , nearest to midspan (or the heaviest load).
- If Wcr is placed to one side of midspan C, the resultant of the load system R shall be on the other side of C; and Wcr and R shall be equidistant from C.
- Now the absolute maximum bending moment will occur under Wcr .
- If Wcr and R coincide, the absolute maximum bending moment will occur at midspan.

What is the absolute maximum bending moment due to a moving udl longer than the span of a simply supported beam?

- When a simply supported beam is subjected to a moving udl longer than the span, the absolute maximum bending moment occurs when the whole span is loaded.
- Mmax max = $wl^2/ 8$

State the location of maximum shear force in a simple beam with any kind of loading.

- In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

What is meant by maximum shear force diagram?

- Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions.
- When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram.
- This diagram yields the 'design shear' for each cross section.

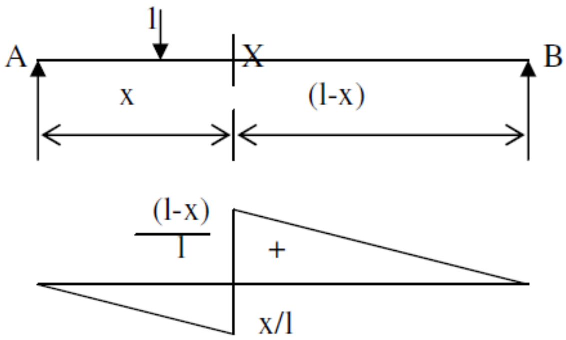
What is meant by influence lines?

- An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

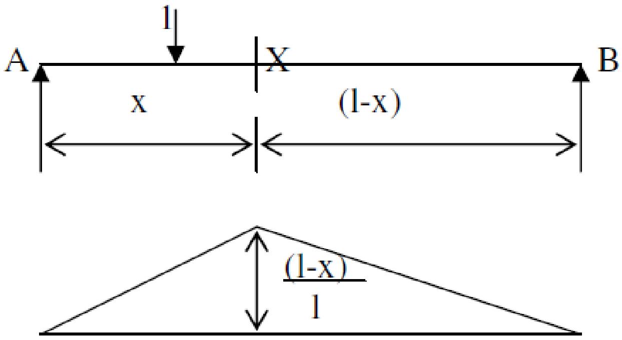
What are the uses of influence line diagrams?

- Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and
- Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

Draw the influence line diagram for shear force at a point X in a simply supported beam AB of span 'l' m.



Draw the ILD for bending moment at any section X of a simply supported beam and mark the ordinates.



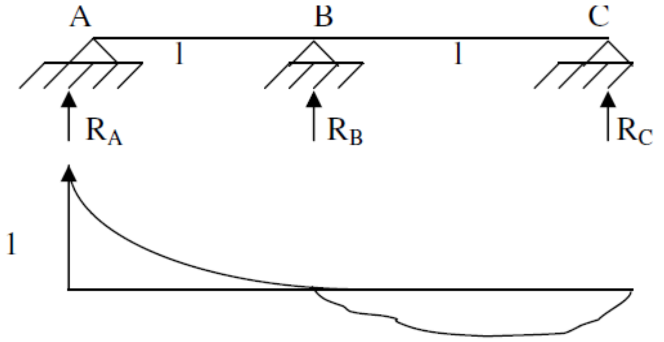
What do you understand by the term reversal of stresses?

- In certain long trusses the web members can develop either tension or compression depending upon the position of live loads.
- This tendancy to change the nature of stresses is called reversal of stresses.

State Muller-Breslau principle.

- Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,
- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity.
- The resulting displacements in the structure are the influence line ordinates sought.

State Maxwell-Betti's theorem.



- In a linearly elastic structure in static equilibrium acted upon by either of two systems of external forces, the virtual work done by the first system of forces in undergoing the displacements caused by the second system of forces is equal to the virtual work done by the second system of forces in undergoing the displacements caused by the first system of forces.
- Maxwell Betti's theorem helps us to draw influence lines for structures.

What is the necessity of model analysis?

- When the mathematical analysis of problem is virtually impossible.
- Mathematical analysis though possible is so complicated and time consuming that the model analysis offers a short cut.
- The importance of the problem is such that verification of mathematical analysis by an actual test is essential.

Define similitude.

- Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- Geometric similitude is similarity of form
- Kinematic similitude is similarity of motion
- Dynamic and/or mechanical similitude is similarity of masses and/or forces.

State the principle on which indirect model analysis is based.

- The indirect model analysis is based on the Muller Breslau principle.
- Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.,
- To get the influence line for any force quantity,
    - (i) remove the resistant due to the force,
    - (ii) apply a unit displacement in the direction
    - (iii) plot the resulting displacement diagram.
- This diagram is the influence line for the force.

What is the principle of dimensional similarity?

- Dimensional similarity means geometric similarity of form.
- This means that all homologous dimensions of prototype and model must be in some constant ratio.

What is Begg's deformeter?

- Begg's deformeter is a device to carry out indirect model analysis on structures.
- It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model.
- In addition, it provides facility to measure accurately the consequent displacements all over the model.

Name any four model making materials.

- Perspex,
- plexiglass,
- acrylic,
- plywood,
- sheet araldite
- bakelite
- Micro-concrete,
- mortar and plaster of paris

What is 'dummy length' in models tested with Begg's deformeter.

- Dummy length is the additional length (of about 10 to 12mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

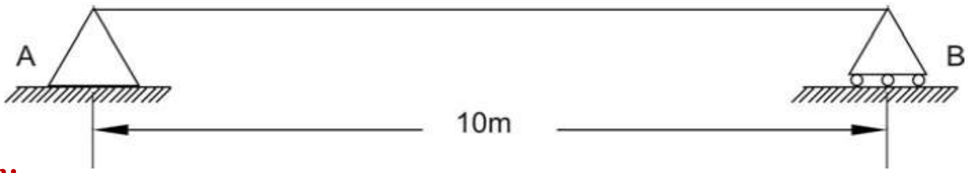
What are the three types of connections possible with the model used with Begg's deformeter.

- Hinged connection
- Fixed connection
- Floating connection

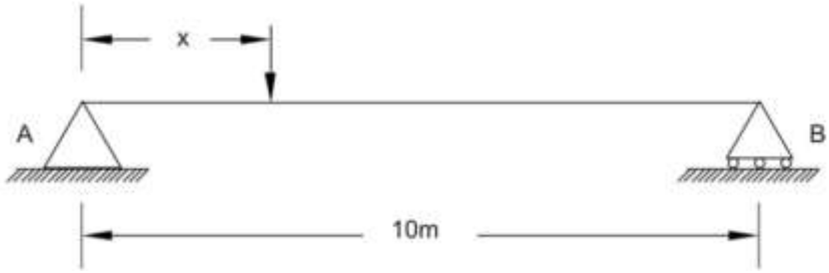
What is the use of a micrometer microscope in model analysis with Begg's deformeter.

- Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg's deformeter.

Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Figure



Solution:



- A unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A.
- Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows

$$\Sigma MA = 0:$$

$$R_B \times 10 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.25$$

- Similarly, the load can be placed at 5.0, 7.5 and 10 m away from support A and reaction R_B can be computed and ta bulated as given below.

X	**R_B**
0	0
2.5	0.25
5	0.5
7.5	0.75
10	1

- Graphical representation of influence line for RB is shown in Figure

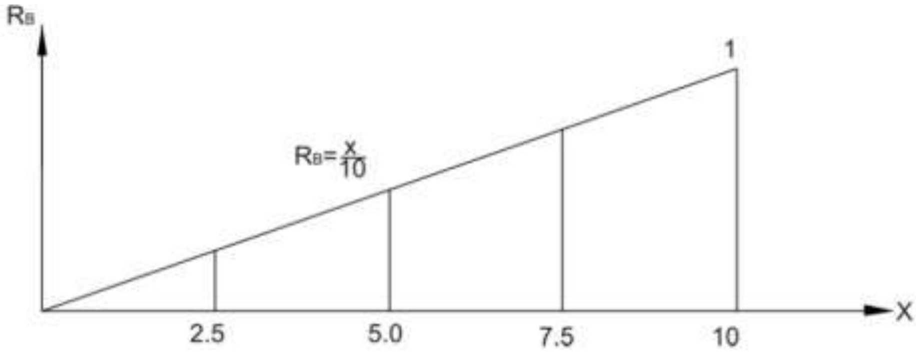
Influence Line Equation:

- When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction RB can be written as

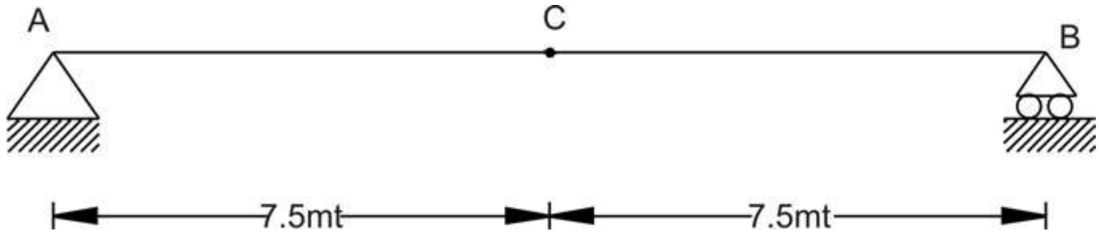
$$\Sigma MA = 0:$$

$$RB \times 10 - x = 0 \Rightarrow RB = x/10$$

Influence line for reaction R_B.

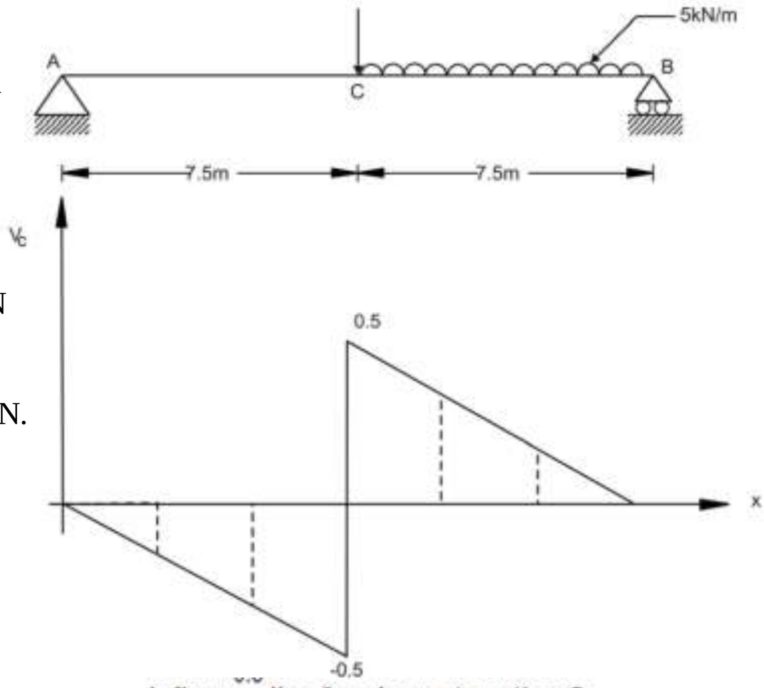


Find the maximum positive live shear at point C when the beam as shown in figure, is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.



Concentrated load:

- the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C.
- Our aim is to find positive live shear and hence, we will put 10 kN just after C.
- In that case, Vc = 0.5 x 10 = 5 kN.



Influence line for shear at section C.

UDL:

- the maximum positive live shear force at C willbe when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

Vc = [0.5 x (15 –7.5) (0.5)] x 5 = 9.375

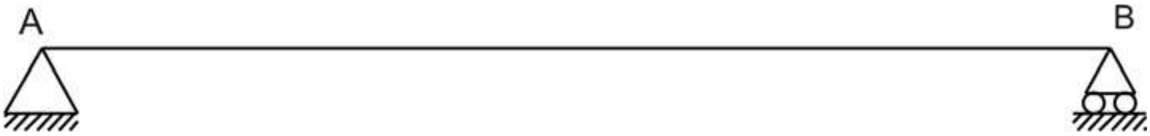
Total maximum Shear at C:

(Vc) max = 5 + 9.375 = 14.375.

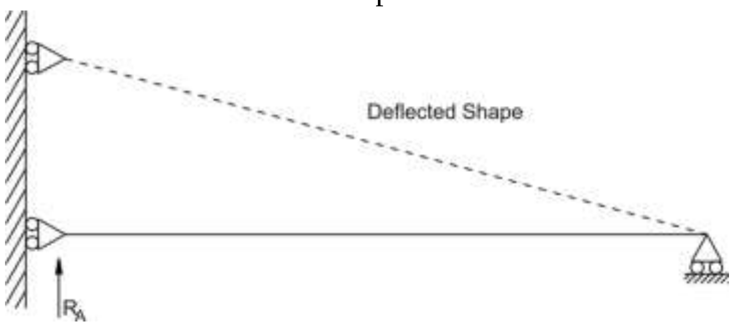
Muller Breslau Principle for Qualitative Influence Lines

- In 1886, Heinrich Müller Breslau proposed a technique to draw influence lines quickly.
- The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

Procedure:

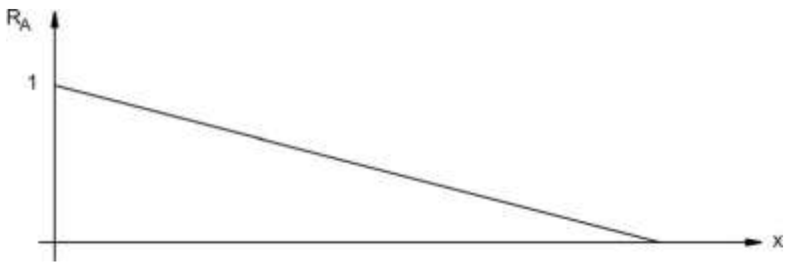


- First of all remove the support corresponding to the reaction and apply a force in the positive direction that will cause a unit displacement in the direction of R_A



Deflected shape of beam

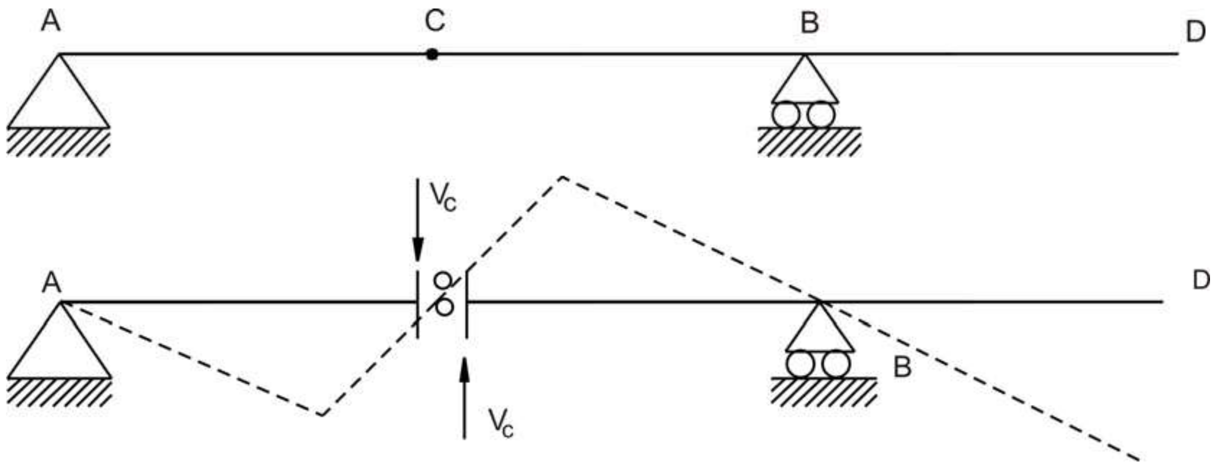
- The resulting deflected shape will be proportional to the true influence line for the support reaction at A.



Influence line for support reaction A

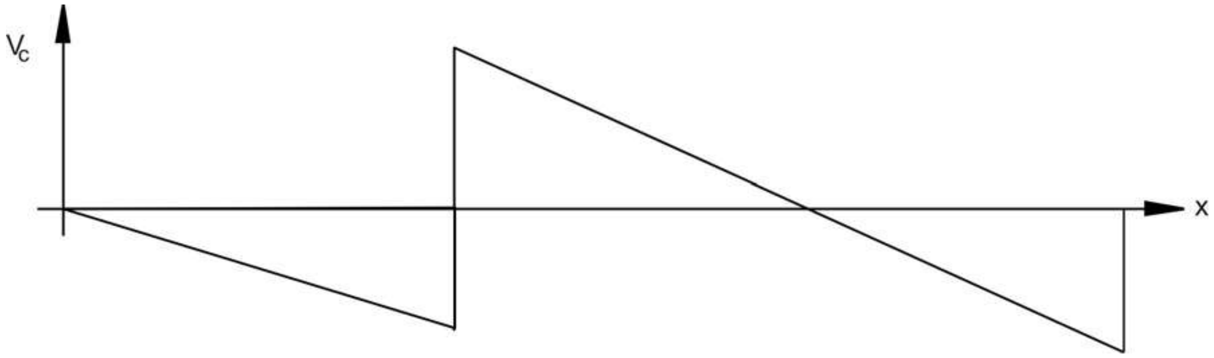
- The deflected shape due to a unit displacement at A is shown in above Figure:1 and matches with the actual influence line shape as shown in Figure 3.
- Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Overhang beam



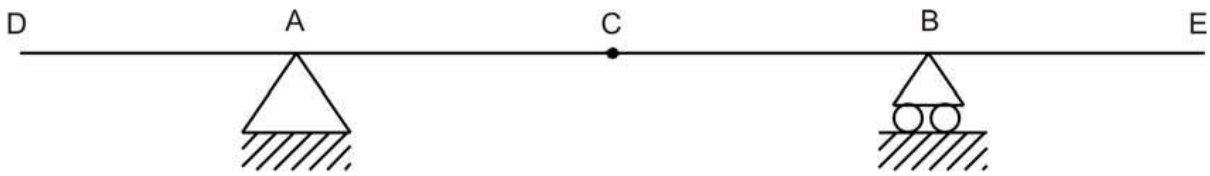
Deflected shape of beam

- Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C.
- The resultant deflected shape is shown above Figure. Again, note that the deflected shape is linear.



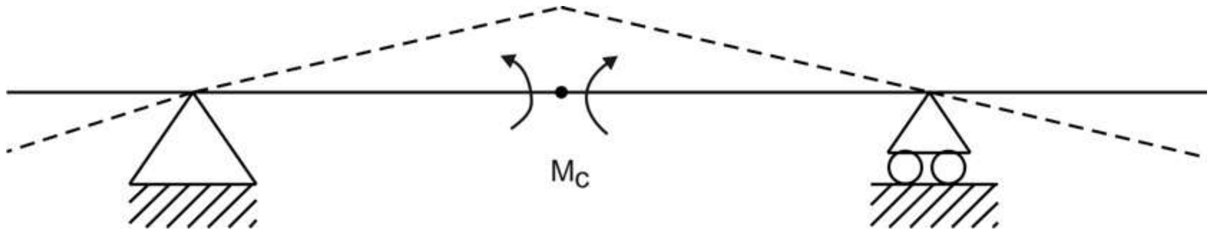
Influence line for shear at section C

Overhang beam - 2

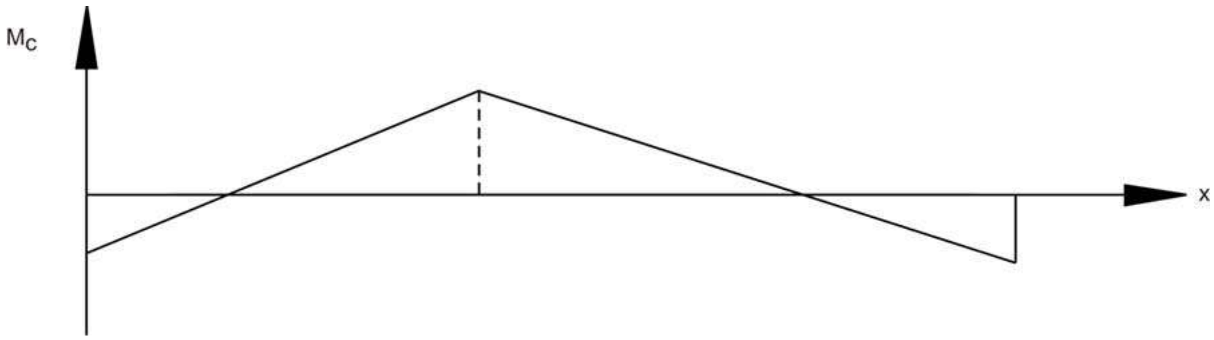


Beam structure

- To construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C.
- Now apply moment in the positive direction that will cause a unit rotation in the direction of Mc.
- The deflected shape due to a unit rotation at C is shown in Figure and matches with the actual shape of the influence line as shown in Figure 3.



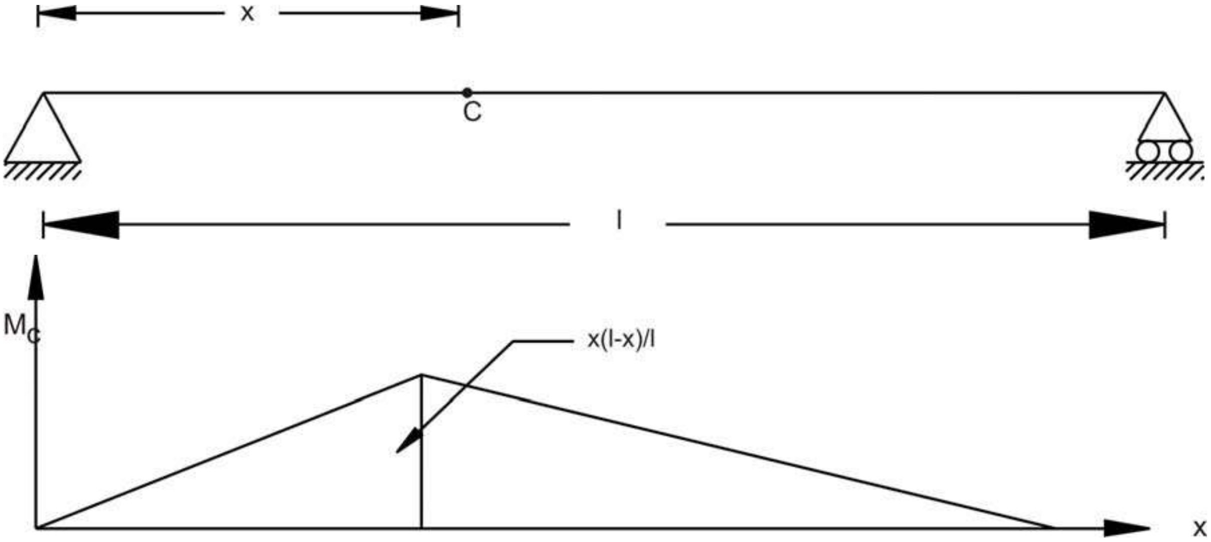
Deflected shape of beam



Influence line for moment at section C

Maximum shear in beam supporting UDLs

UDL longer than the span



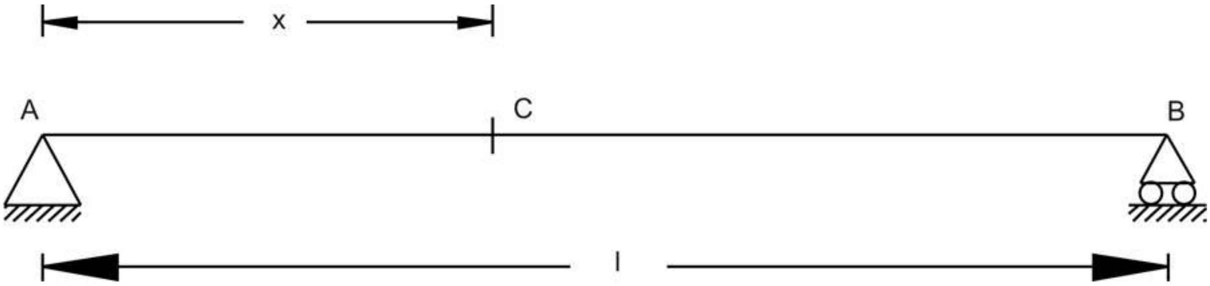
Influence line for moment at section C

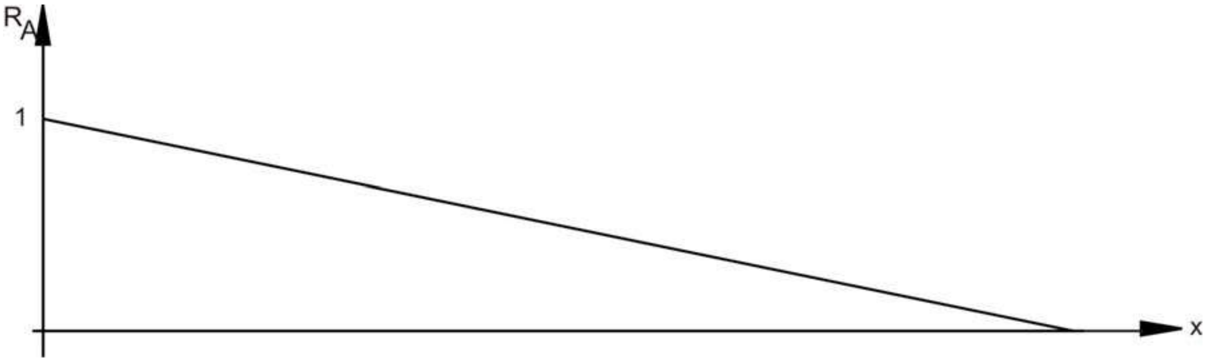
$$w \times \frac{1}{2} \times l \times \frac{x(l-x)}{l} = -\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

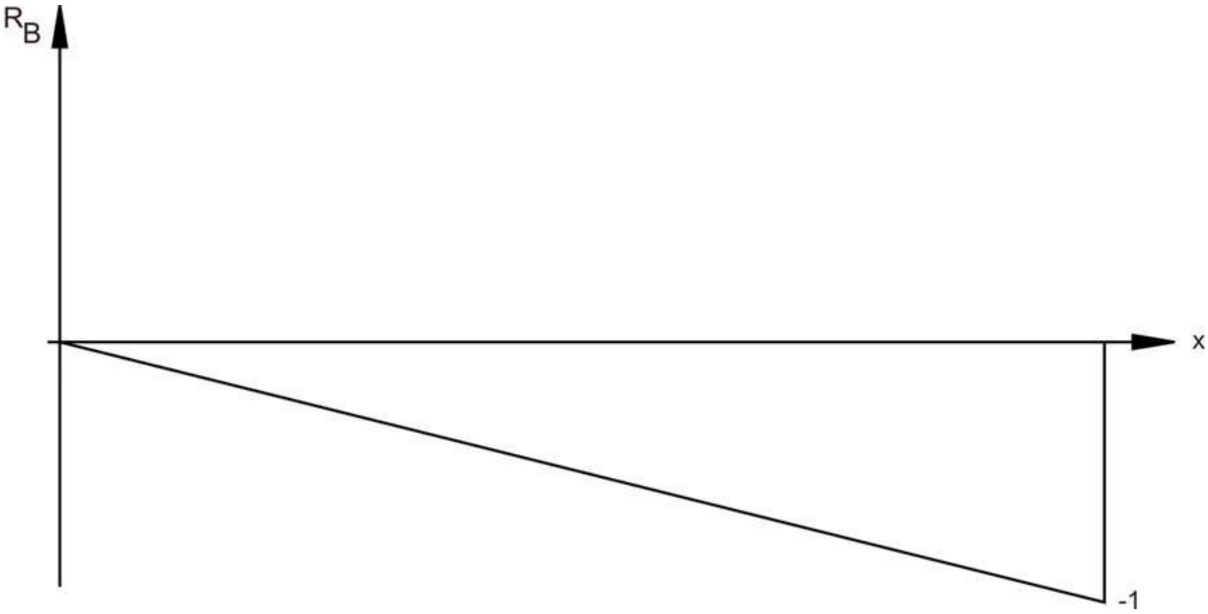
$$\frac{w \times \frac{l}{2} \times \frac{l}{2}}{2} = \frac{wl^2}{8}$$

UDL longer than the span

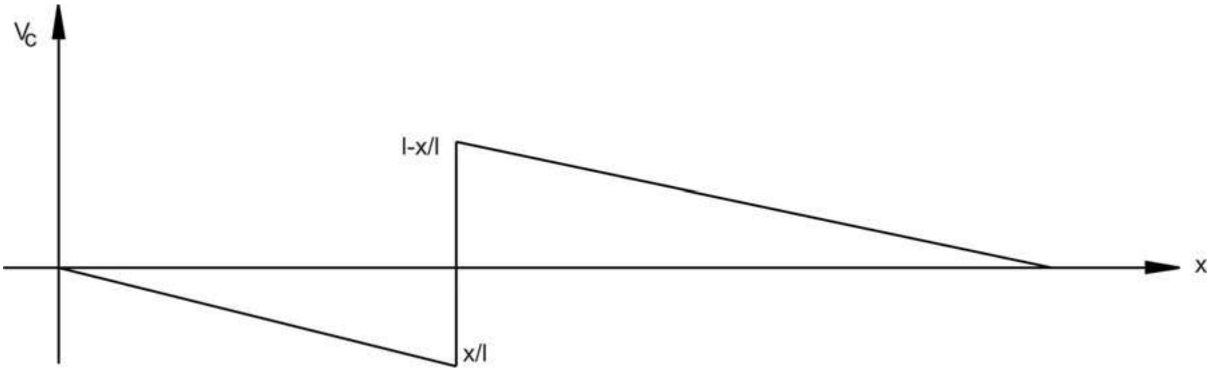




Influence line for support reaction at A



Influence line for support reaction at B



Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Maximum negative shear is given by

$$= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$$

Maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l}\right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$