FREQUENCY DOMAIN SPECIFICATIONS

The translation of time domain specifications into desired locations of pair of dominant closed loop poles in the z-plane is useful if the design is to be carried out by using the root locus plots. The use of frequency response plots necessitates the translation of time domain specifications in terms of frequency response features. All the frequency domain methods of continuous system can be extended for the analysis and design of digital control systems.

Consider the system shown in fig below The closed loop transfer function of the sampled data system is

$$\frac{C(z)}{R(z)} = \frac{G_{h0}G(z)}{1+G_{h0}G(z)}$$

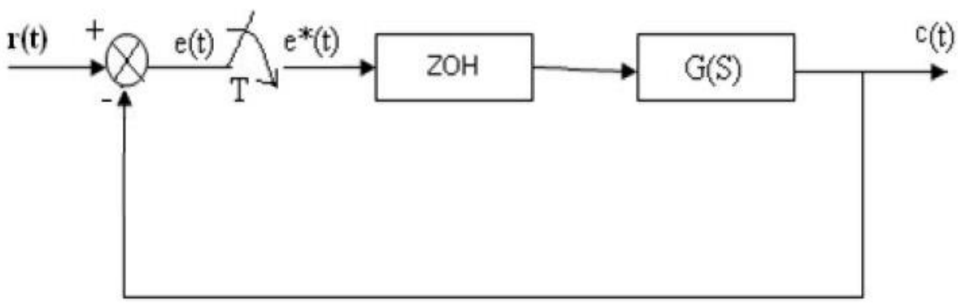


Fig: A unity feedback discrete time system

Just as in case of continuous time systems, the absolute and relative stability of the closed loop discrete time system can be investigated by making the frequency response plots of $G_{h0}G(z)$. The frequency response plots of $G_{h0}G(z)$ are obtained by setting $z = e^{j\omega t}$ and letting ω vary from $-\omega_s/2$ to $\omega_s/2$. This is equivalent to mapping the unit circle in the z-plane into $G_{h0}G(e^{j\omega t})$ plane. Since the unit circle in the z-plane is symmetrical about real axis, the frequency plot of $G_{h0}G(e^{j\omega t})$ will also be symmetrical about real axis, so that only the portion that corresponds to ω=0 to $\omega = \omega_s/2$ needs to be plotted.

A typical curve of closed loop frequency response i.e, $\frac{C}{R}\left(e^{jwt}\right) = \frac{G_{h0}G\left(e^{jwt}\right)}{1+G_{h0}G\left(e^{jwt}\right)}$

The amplitude ratio and the phase angle will approximately idle $1\angle 0$ for some range of low frequency but will deviate high for high frequencies.

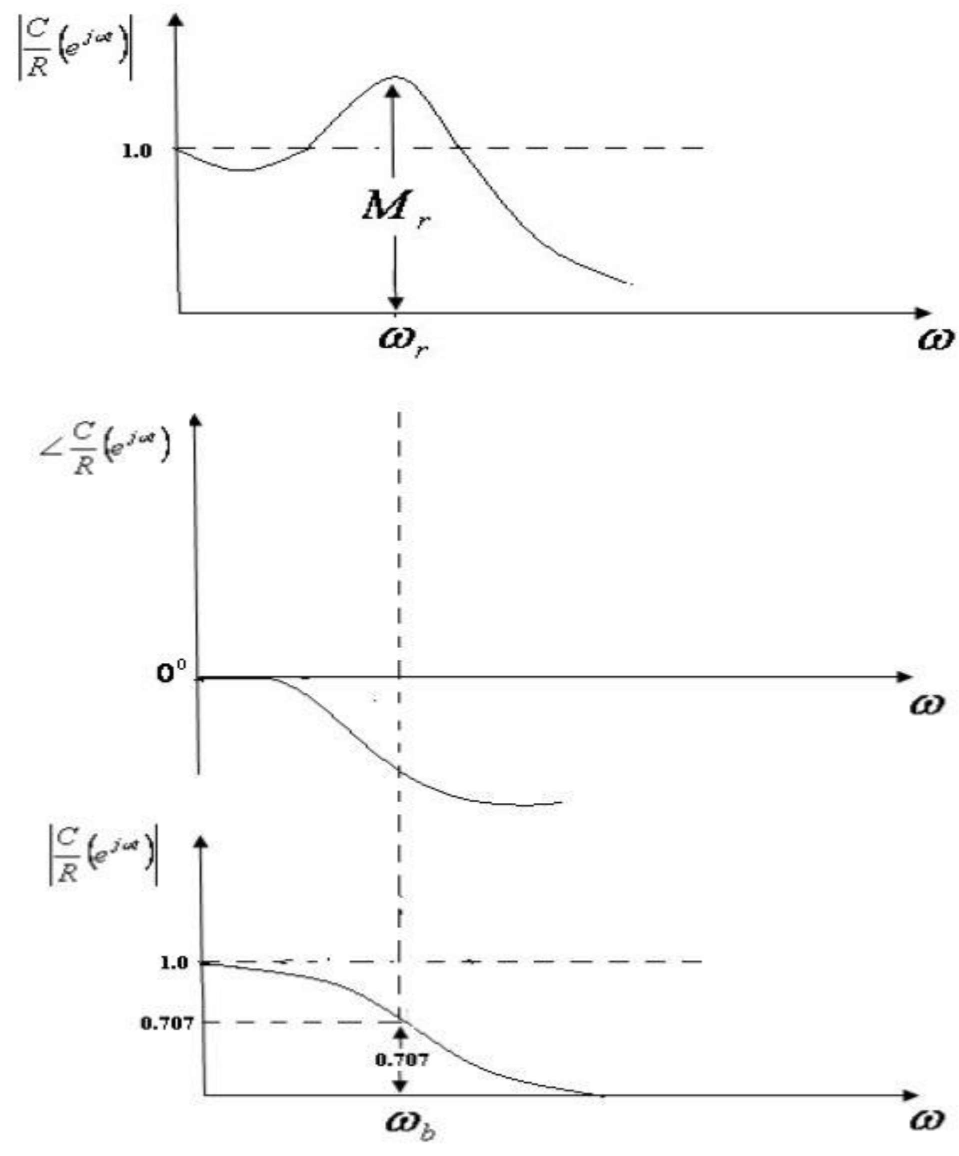


Fig : Closed loop frequency response criteria

Resonant peak (M_r): It is the maximum value of the magnitude of the closed loop frequency as shown in figure . The height M_r (resonant peak) of the peak is a relative stability criterion, the higher the peak the poorer the relative stability. Many systems are designed to exhibit a resonant peak in the range of 1.2 to 1.4

Resonant frequency (ω_r): The frequency at which the resonant peak occurs in a closed loop frequency response is called resonant frequency; it is a speed of response criterion, the higher the ω_r the faster the system.

Band width (ω_d): It is defined as the range of frequencies over which the system will respond satisfactorily. It can also be defined as the range of frequencies in which the magnitude response is flat in nature. (or) Band width is the frequency at which the amplitude ratio has dropped to 0.707 times its zero frequency value. It can, of course, be specified even if there is a peak. It indicates the speed of the response.

Two open loop performance criteria are in common use to specify relative stability. These are gain margin and phase margin. A typical curve of $G_{h0}G(e^{j\omega t})$ the open loop frequency response is shown on polar plane

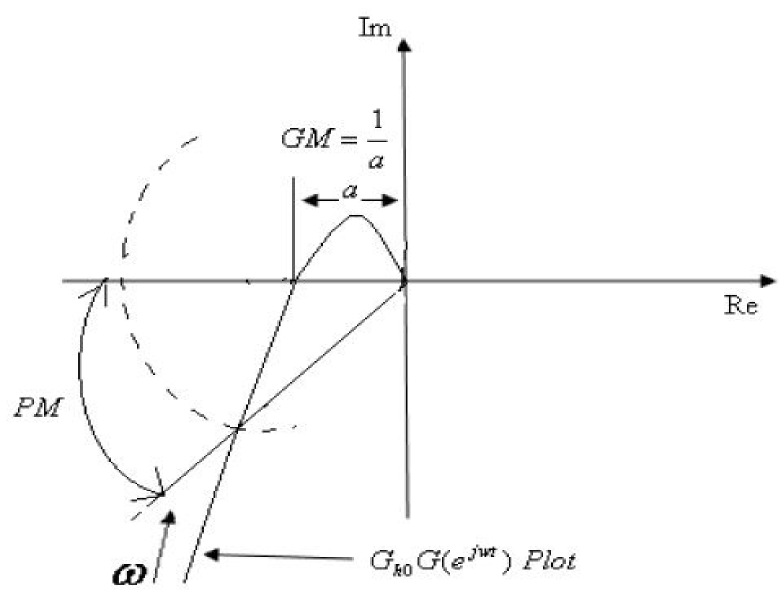


Fig : Phase margin and Gain Margin

Gain margin (GM): Gain margin is the multiplying factor by which the steady state gain of $G_{h0}G(e^{j\omega t})$ would be increased so as to put the system on the edge of instability.

Phase margin (PM): Phase margin is the number of degrees of additional phase lag required to drive the system to the edge of instability.

System is said to be stable when PM and GM are positive and unstable when PM and GM are negative now when the system is on edge of stability i.e marginally stable in nature then the GM and PM are zero. This is possible when $\omega_{gc} = \omega_{pc}$

The translation of time domain specifications in terms of frequency response features is carried out by using the explicit correlations for second order system. The following correlations are valid approximations for higher order systems dominated by a pair of complex conjugate poles.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$\omega_b = \omega_n\left[1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}\right]^{1/2}$$

$$PM = \tan^{-1}\left\{\frac{2\xi}{\left[\sqrt{1-2\xi^2+4\xi^4}\right]^{1/2}}\right\}$$

FREQUENCY RESPONSE PLOTS

Frequency domain analysis and design posses a wealth of graphical and semi-graphical techniques that can be applied to linear time invariant control systems. Historically, the analysis and design of continuous-data control systems in the frequency domain have been well developed, and practically all these methods can be extended to digital control systems. Such well known methods as the Nyquist criterion for stability analysis, the Bode plot and the Nichols chart can all be extended to analysis design of control systems without complications.

The study of digital control systems in the frequency domain essentially relies on the extension of all the existing techniques devised for the analysis of continuous data systems. Some of the well-known methods are described as follows.

(i) The Nyquist Plot:

The Nyquist plot of a transfer function, usually the loop transfer function GH(z), is a mapping of the Nyquist path in the z-plane on to the GH(z) plane which is in polar coordinates. Thus the Nyquist plot is known as polar plot. Absolute and relative stabilities of the closed loop digital control systems can be determined from the Nyquist plot of GH (z)

(ii) The Bode Diagram:

The bode diagram is a plot of the amplitude in dB and the phase angle of a transfer function, usually the open loop transfer function GH(z) as a function of frequency ω. The Bode diagram may be used to investigate the absolute and relative stabilities of a closed loop digital control system.

(iii) The Gain Phase Plot:

The gain-phase plot of a open loop transfer function of a control system is a plot of amplitude in dB versus phase in degrees. The plot can be used to determine absolute and relative stabilities of the closed loop system. When the gain-0phase plot of GH(z) is super imposed on the Nicholas chart, relative stability and information on the closed loop frequency response can be obtained.

DESIGN OF DISCRETE-DATA SYSTEMS

The design problems encountered in discrete-data control systems are essentially similar to those found in the design of continuous data control systems. Basically, a process or plant needs to be controlled so that its output will behave according to some prescribed performance specifications. In the conventional design approach, we decide at the outset that there should be feedback from the outputs to the reference inputs, so that errors can be formed between these signals for control efforts. Then, in general, we find that a controller is needed to operate on the error signals in such a way that the design specifications are satisfied by the outputs.

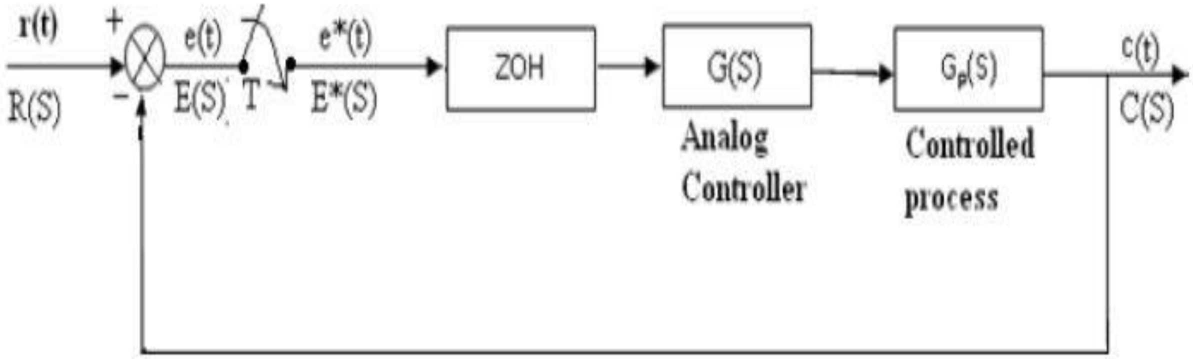


Fig : Digital control system with cascade controller

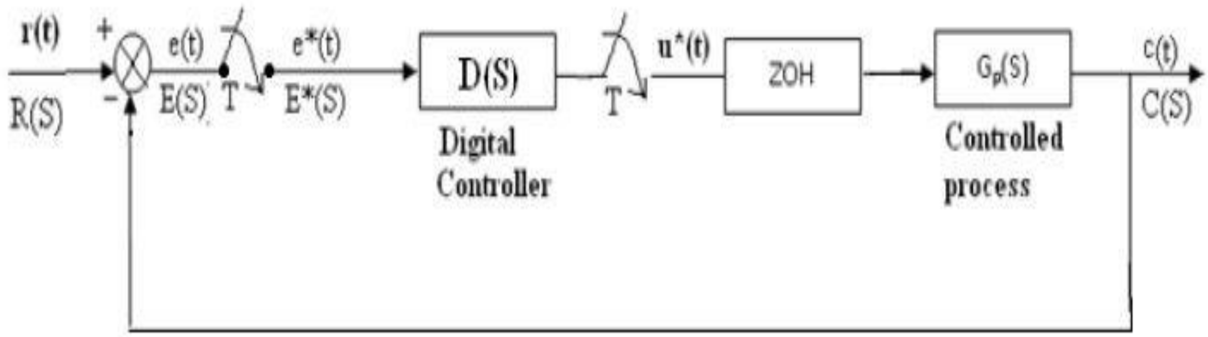


Fig : Digital control system with cascade digital controller

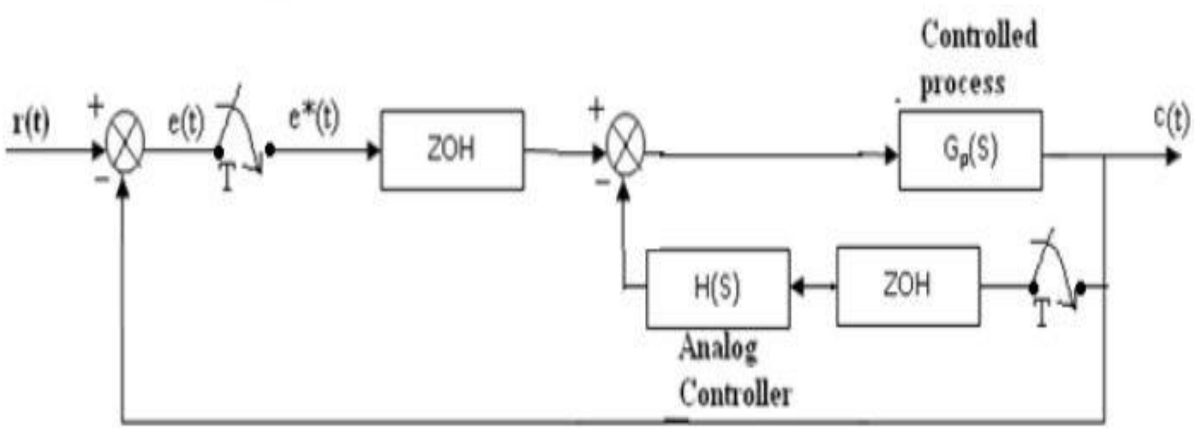


Fig: Digital control system with analog controller in minor feedback path

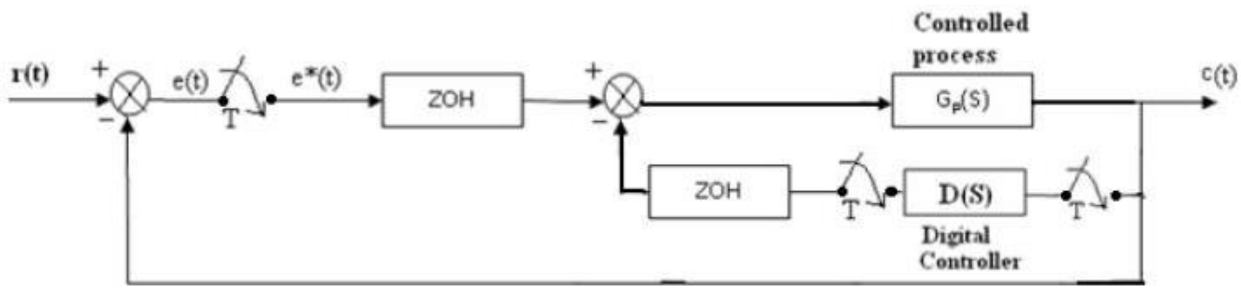


Fig : Digital control system with digital controller in minor feedback path

Block - diagram of a sampled-data system, in which the controller is analog. The sampler may represent the fact that digital or sampled data exist at the input and feedback channels, due to the use of digital transducers. In this case, a continuous-data controller is selected to operate on the sampled signal after it is decoded and smoothened out by the data hold and the pervious classical case of a digital control system, in which a digital controller is located in the forward path. The digital controller operates on the digital signal $e^*(t)$, which is represented as the output of a sampler, and outputs the digital signal $u^*(t)$, which in turn is filtered by the usual data hold before being applied to controlled process.

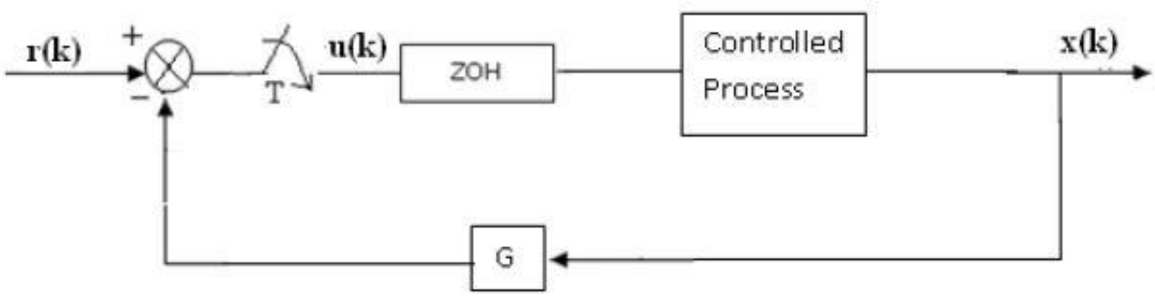


Fig : Digital control system with state feedback

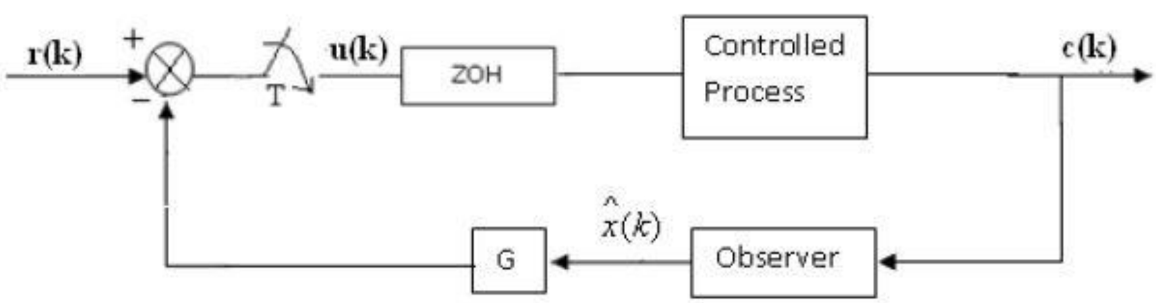


Fig : Digital control system with state feedback and observer

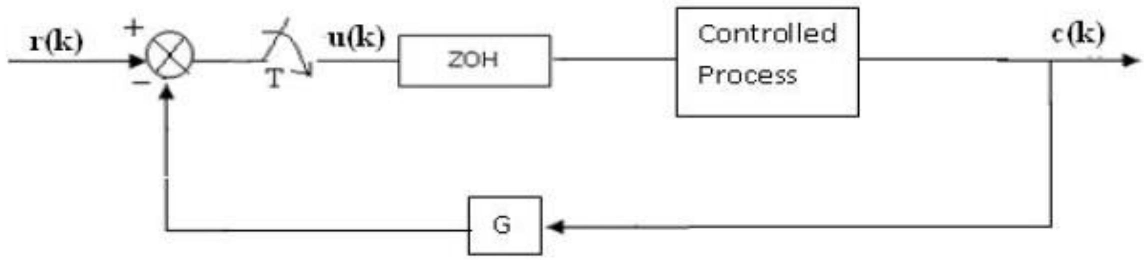


Fig : Digital control system with output feedback

THE DIGITAL PID CONTROLLER

One of the most widely used controllers in the design of continuous-data control system is the proportional-integral-derivative (PID) controller. Fig below shows the block diagram of a continuous-data PID controller acting on error signal e(t). The proportional control simply multiplies e(t) by a constant K_p, the integral control multiplies the time integral of e(t) by K_I and the derivative control generates a signal equal to K_D times the time derivative of e(t). The integral control is used to reduce the steady-state error and the derivative control reduce the over shoots and oscillations in the time response and thus improves the transient response of the system.

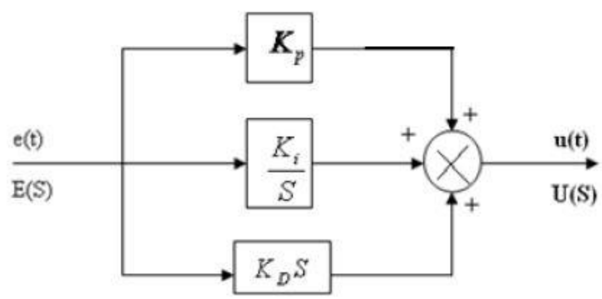


Fig : A Continuous-data PID Controller

The same principle of PID controller can be applied to digital control. In digital control; the proportional control is still implemented by a proportional constant K_p. In general, there are a number of ways of implementing integration and derivatives digitally, and generally the rectangular integration schemes are used for this purpose. The transfer functions of these integration schemes including the proportional constant K_I are summarized as follows:

Backward Rectangular Integration

$$D_I(z) = K_I \frac{T}{z-1}$$

Forward Rectangular Integration

$$D_I(z) = K_I \frac{Tz}{z-1}$$

Bilinear-transformation Integration

$$D_I(z) = K_I \frac{T}{2}\left(\frac{z+1}{z-1}\right)$$

The most common method of approximating the derivative of $e(t)$ at t=T that results in a physically realizable transfer function is

$$\left.\frac{de(t)}{dt}\right|_{t=T} = \frac{e(kT) - e[(k-1)T]}{T}$$

By taking the z-transforms an both sides and including the proportional constant K_D, we have the transfer function of the digital derivative controller as

$$D_I(z) = K_D\left(\frac{z-1}{Tz}\right)$$

The block diagram of the digital PID controller is as shown in fig.(7.16).

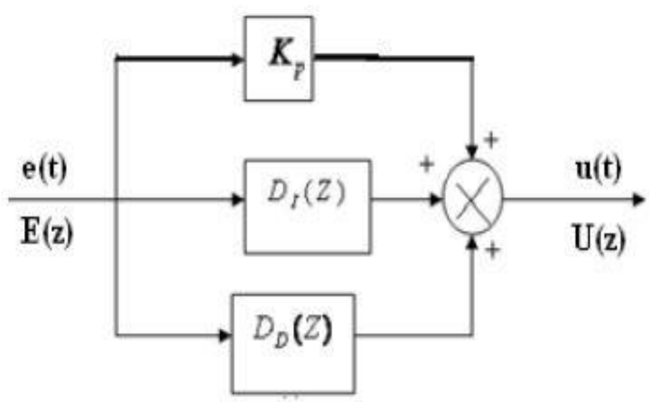


Fig: A Digital PID Controller

Thus the PID controller is represented by the following transfer function

$$D(z) = K_P + D_I(z) + D_D(z)$$

Using the three rectangular integration schemes, the transfer function of the digital PID controller are summarized as follows:

Backward Rectangular Integration

$$D(z) = K_P + K_I \frac{T}{z-1} + K_D\left(\frac{z-1}{Tz}\right)$$

$$= \frac{K_P Tz(z-1) + T^2 K_I z + K_D (z-1)^2}{Tz(z-1)}$$

$$= \frac{(K_P T + K_D)z^2 + (K_I T^2 - K_P T - 2K_D)z + K_D}{z(z-1)T}$$

Forward rectangular integration

$$D(z) = K_P + K_I \frac{T}{z-1} + K_D\left(\frac{z-1}{Tz}\right)$$

$$= \frac{K_P Tz + K_I T^2 z^2 + K_D (z-1)^2}{Tz(z-1)}$$

$$= \frac{(K_P T + K_D + K_I T^2)z^2 - (K_P T + 2K_D)z + K_D}{z(z-1)T}$$

Bilinear transformation integration

$$D(z) = K_P + K_I \frac{T}{2}\frac{z+1}{z-1} + K_D\left(\frac{z-1}{Tz}\right)$$

$$= \frac{K_P 2Tz(z-1) + T^2 z(z+1) + 2K_D (z-1)^2}{2Tz(z-1)}$$

$$= \frac{(2K_P T + 2K_D + K_I T^2)z^2 - (K_I T^2 - 2K_P T + 4K_D)z + 2K_D}{2z(z-1)T}$$

Thus the digital PID controller has a pole at z=0 and one at z=1 . There are two zeros which can be real or in complex conjugate pairs.

The digital PD controller

Depending on the design requirements, frequently only the proportional and derivative components of the PID controller and needed. Setting $K_I = 0$ in any one of the transfer function we get the transfer function of the digital PI controller as

$$D(z) = \frac{(K_P T + K_D)z - K_D}{zT}$$

Thus, the digital PD controller has a pole at z=0 and a zero at $\frac{K_D}{K_P T + K_D}$ which lies on the positive real axis inside the unit circle.

The digital PI controller

Under certain conditions using only the proportional and integral components of the PID controller is adequate for design purposes. Setting $K_D = 0$ in the transfer functions

For the three cases of integration, we get the following results

Backward-Rectangular Integration:

$$D(z) = \frac{K_p z - (K_p - K_I T)}{z - 1}$$

Forward- Rectangular Integration:

$$D(z) = \frac{(K_p + K_I T)z - K_P}{z - 1}$$

Bilinear- Transformation Integration:

$$D(z) = \frac{(2K_p + K_I T)z + (K_I T - 2K_P)}{2(z - 1)}$$

DESIGN OF DIGITAL CONTROL SYSTEMS WITH DIGITAL CONTROLLERS THROUGHT BILINEAR TRANSFORMATION

Here we consider the design of a digital control system with a digital controller using the frequency domain technique. This type of design problem is generally simpler to carry out than the design of discrete data system with continuous data controllers. The reason is because the transfer function of the digital controller is isolated from that of the controlled process, so that the effects of varying the controller parameters may be investigated by means of bode diagram using either the r-transformation $(z = \frac{1+r}{1-r})$ or the w- transformation $z = \frac{2/T + w}{2/T - w}$

The principle of design is obtained below with reference to the block diagram using the r-transformation.

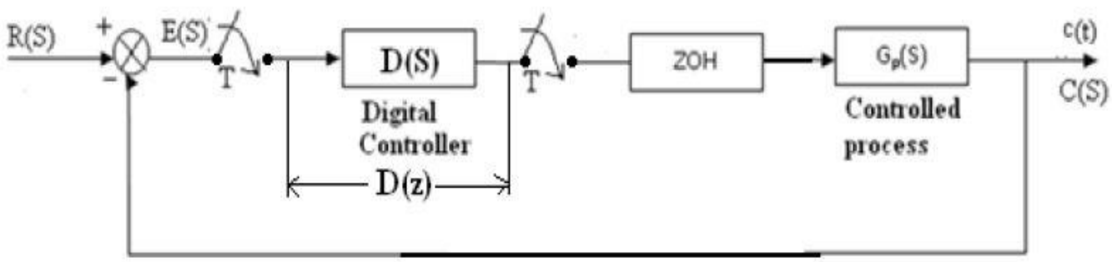


Fig : A control system with digital controller

1. Evaluate the Z-transform of the ZOH and controlled process combination $G_{h0}G(z)$. Apply the r-transformation, $z = \frac{1+r}{1-r}$, to obtain $G_{h0}G(r)$.
2. Construct the bode diagram of $G_{h0}G(r)$ in magnitude (dB) and phase (degree) verses $\boldsymbol{\omega_r}$.

Transfer the data from Bode plot to the Nichols chart if necessary. Determine the performance characteristics of the uncompensated system by finding the gain margin, phase margin, bandwidth, resonant peak and resonant frequency from the Bode plot and Nichols chart.

3. If the system needs compensation, the open loop transfer function of the system with digital controller becomes D(z) $G_{h0}G(z)$ or in the r-domain, D(r) $G_{h0}G(r)$. The digital controller transfer function D(r) is to be determined so that the desired system performance specifications are satisfied. The selection of D® may follow the design principle of continuous-data system, which involves a trial-and-error procedure and, to some extent, is based on the experience and imagination of the designer.

4. Once D(r) is determined; D (z) is obtained by substituting $r = \frac{z-1}{z+1}$ in D(r). The final step in the design involves the realization of D (z) by one of the digital programming methods. If D(z) is to be implemented by a micro processor or DSP, then the designer should be aware of the limitations and constrains of these devices and take them into considerations when carried out the design .

   Note: The prior requirement on the design of the digital controller is that the transfer function D(z) be physically realizable. The condition of physical realizability implies that no output signal of the system will appear before an input signal is applied.

Introduction to compensation

In building a control system, we know that proper modification of the plant dynamics may be a simple way to meet the performance specifications. This, however, may not be possible in many practical situations because the plant may be fixed and not modifiable. Then we must adjust parameters other than those in the fixed plant. In order to achieve the desired system response, it is possible to adjust the system parameters but it is often not enough. It is then required to reconsider the structure of the system and redesign the system.

The design problems, therefore, become those of improving system performance by insertion of a compensator.

Compensator: A compensator is an additional component or circuit that is inserted into a control system to equalize or compensate for a deficient performance.

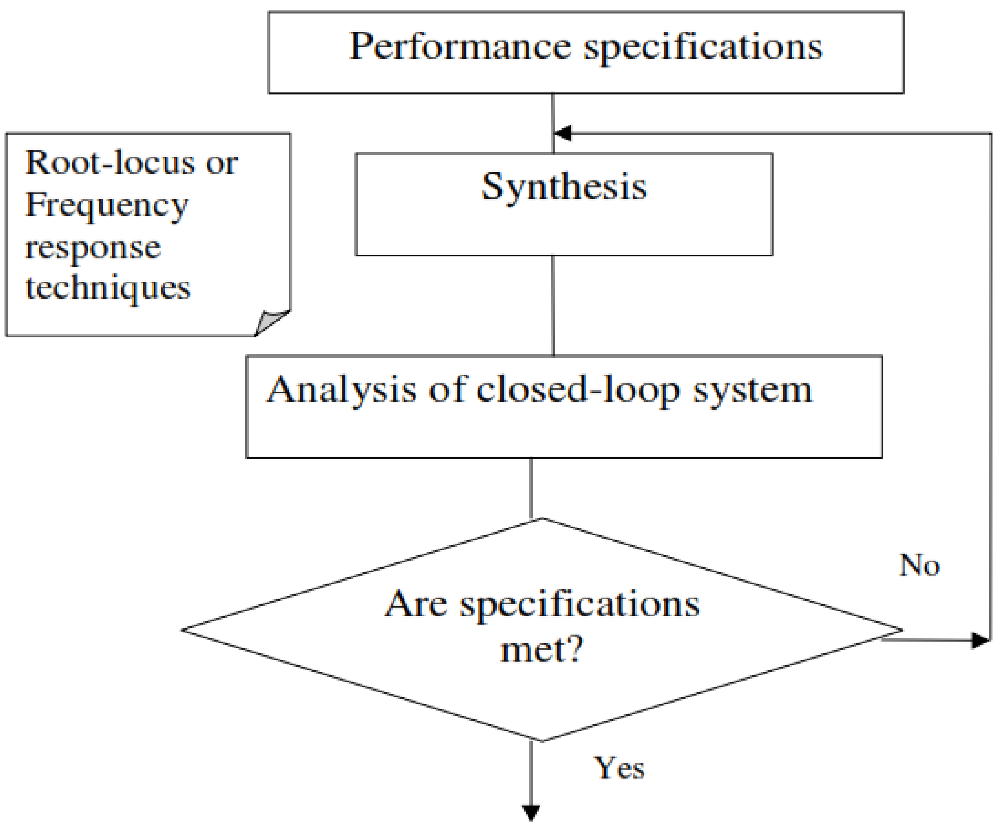
Necessities of compensation

A system may be unsatisfactory in : 1.Stability. 2. Speed of response. 3. Steady-state error.

Thus the design of a system is concerned with the alteration of the frequency response or the root locus of the system in order to obtain a suitable system performance.

Flow Chart of the design approach

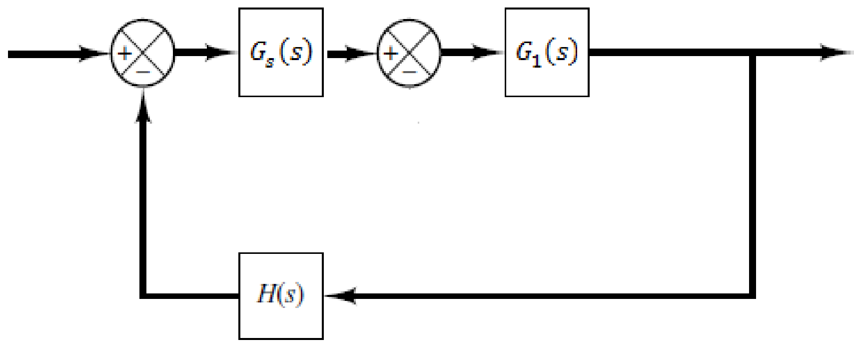
- Trial and error approach to design



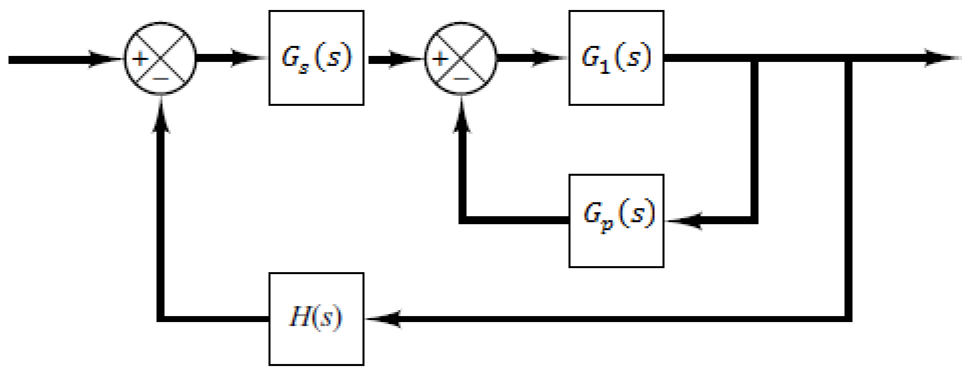
Compensator Configurations

Compensation schemes commonly used for feedback control systems are

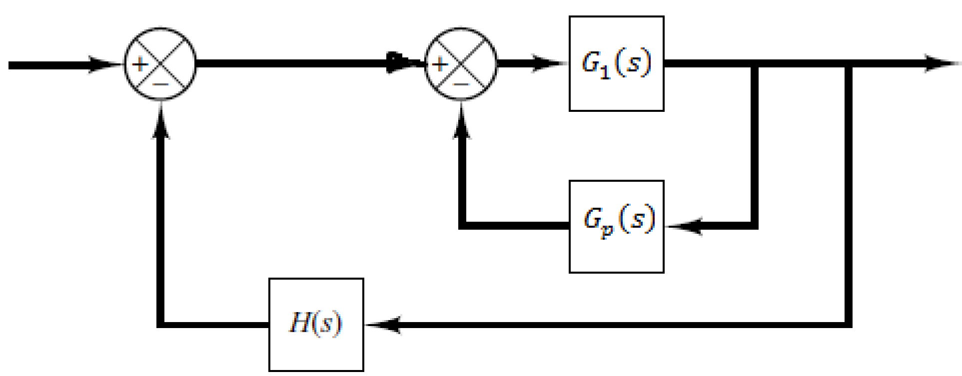
1. Series Compensation



2. Series-Parallel Compensation



3. Parallel Compensation



- Among the many kinds of compensators, widely employed compensators are the

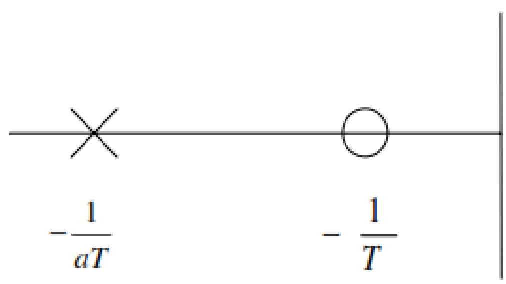
1. lead compensators will improves the transient response
2. lag compensators will improves the steady-state performance
3. lag–lead compensators will improves the both

- lead compensators
  - If a sinusoidal input is applied to the input of a network, and the steady-state output (which is also sinusoidal) has a phase lead, then the network is called a lead network.

$$G_D(w) = K_D \frac{1+Tw}{1+\alpha Tw}$$

Where, T>0 and 0< α<1

- Poles and zeros of the lead compensator:

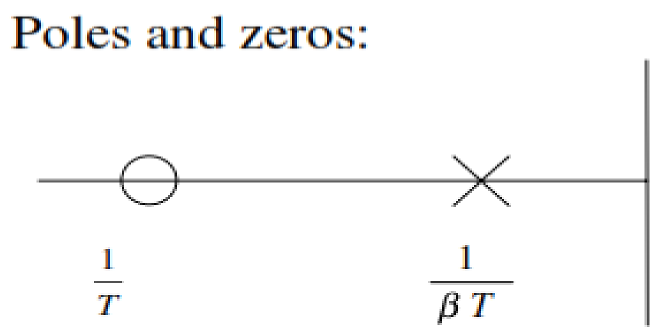


lag compensators

If the steady-state output has a phase lag, then the network is called a lag network.

$$G_D(w) = K_D \frac{1+Tw}{1+\boldsymbol{\beta}Tw}$$

Where, $T>0$ and $\boldsymbol{\beta}>1$



lag–lead compensators

In a lag–lead network, both phase lag and phase lead occur in the output but in different frequency regions.

Phase lag occurs in the low-frequency region and phase lead occurs in the high-frequency region.

$$G_D(w) = K_D \frac{1+Tw}{1+\alpha Tw} * K'_D \frac{1+Tw}{1+\boldsymbol{\beta}Tw}$$

Procedure for designing Lead Compensators in the w-plane

Phase Lead Compensation is commonly used for improving stability margins and also increases the system bandwidth. Thus, the system has a faster speed to respond.

Following steps to design lead compensators:

- Step1: First obtain the pulse transfer function G(z) from the given system.

- Step2:Transform G(z) into a transfer function G(w) through the bilinear transformation.

$$z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}$$

that is $\quad G(w)=G(Z)|_{z=\left[1+\left(\frac{wT}{2}\right)\right]/\left[1-\left(\frac{wT}{2}\right)\right]}$

It is important that the sampling period T be chosen properly. A rule of thumb is to sample at the frequency 10 times that of the bandwidth of closed loop system.

- Step3: Substitute w= jv into G(w) and we will plot the bode plot for G(jv).

- Step4: Read from the bode plot diagram the static error constants, the phase margin and the gain margin.

- Step5: By assuming that the low-frequency gain of the discrete-time controller transfer function $G_D(w)$ is unity, determine the system gain by satisfying the requirement for a given static error constant.

- Step6: Then, by using conventional design techniques for continuous-time control systems, determine the poles and zeros of the digital controller transfer function

➢open loop transfer function of designed system is given by $G_D(w)$G(w).

- Step7: Transform the controller transfer function $G_D(w)$ into $G_D(z)$ through the bilinear transformation given by

$$w = \frac{2}{T}\frac{z-1}{z+1}$$

Then

$$G_D(z) = G_D(w)\big|_{w=(\frac{2}{T})(z-1)/(z+1)}$$

Is the pulse transfer function of the digital controller.

Note: The transfer function G(w) is a non minimum phase transfer function.

Hence phase angle curve is drawn by consideration the non minimum phase term.

Example for controller design in w-plane

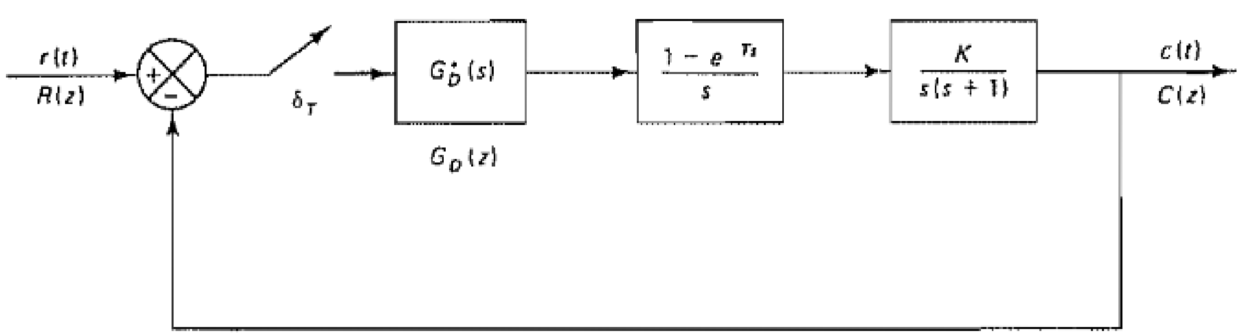
- Consider the digital control system shown in fig. 2. design a digital controller in the w-plane such that

Given specifications

(a). Phase margin is 50°

(b). Gain margin is at least 10db

(c). Static velocity error constant K_v is 2 sec^{-1} and T=0.2 sec.



First, we obtain the pulse transfer function G(z) of the plant that is preceded by the zero-order hold:

$$G(z) = \mathcal{Z}\left[\frac{1 - e^{-0.2s}}{s}\frac{K}{s(s+1)}\right]$$
$$= (1 - z^{-1})\mathcal{Z}\left[\frac{K}{s^2(s+1)}\right]$$
$$= 0.01873\left[\frac{K(z + 0.9356)}{(z-1)(z-0.8187)}\right]$$
$$= \frac{K(0.01873z + 0.01752)}{z^2 - 1.8187z + 0.8187}$$

We transform the G(z) into G(w) by means of bilinear transformation

$$z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}} = \frac{1+0.1w}{1-0.1w}$$

$$G(w) = \frac{K\left[0.01873\left(\frac{1+0.1w}{1-0.1w}\right) + 0.01752\right]}{\left(\frac{1+0.1w}{1-0.1w}\right)^2 - 1.8187\left(\frac{1+0.1w}{1-0.1w}\right) + 0.8187}$$
$$= \frac{K(-0.000333w^2 - 0.09633w + 0.9966)}{w^2 + 0.9969w}$$
$$\doteq \frac{K\left(1 + \frac{w}{300}\right)\left(1 - \frac{w}{10}\right)}{w(w+1)}$$

A simple phase-lead compensator will probably satisfy all requirements.

Now let us assume transfer function of digital controller $G_D(w)$ has unity gain for the low-frequency range and has following form:

$$G_D(w) = \frac{1+Tw}{1+\alpha Tw}, \quad 0< \alpha<1$$

open loop transfer function is

$$G_D(w)\ G(w) = \frac{1+Tw}{1+\alpha Tw}\frac{K(-0.00033w^2-0.09633w+0.9966)}{w^2+0.9969w}$$

The static velocity error constant $\bar{K}_v$ is specified as $2sec^{-1}$. Hence,

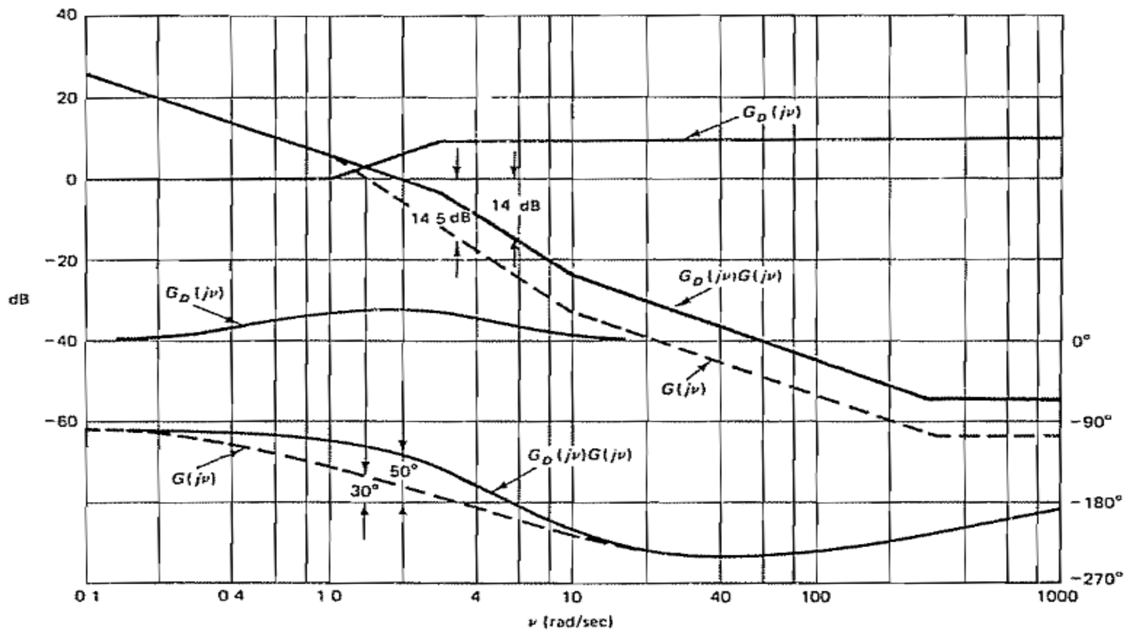
$$K_v = \lim_{w \to 0} wG_D(w)G(w) \longrightarrow \mathrm{K} = 2$$

gain K is thus determined to be 2.

by setting K = 2, we plot the bode diagram of G(w):

$$G(w) = \frac{2(-0.000333w^2 - 0.09633w + 0.9966)}{w^2 + 0.9969w}$$

$$\doteqdot \frac{2\left(1 + \dfrac{w}{300}\right)\left(1 - \dfrac{w}{10}\right)}{w(w + 1)}$$



Bode Diagram for the system designed in given example

- Here the magnitude curves we have used straight-line asymptotes.

- The magnitude and phase angle of G(jv) are shown by dashed curves.

- From the bode diagram (dashed curves)

- phase margin as 30° and gain margin as 14.5dB.

- But given specifications PM = 50°

- GM of at least 10dB and in addition to $K_v = 2$.

- Let us design a digital controller to satisfy these specifications.

- Design of lead compensator : the additional phase lead angle necessary to satisfy this requirement is 20° without decreasing value of K.

- Considering the shift of the gain crossover frequency, we may assume that Φ_m the maximum phase-lead angle required is approximately 28°.

$$\sin \Phi_m = \frac{1+\alpha}{1-\alpha} \quad \Phi_m = 28° \text{ corresponds to } \alpha=0.361$$

- Magnitude of uncompensated system is equal to -20log($1/\sqrt{\alpha}$)

$$-20\log 1.6643 = -4.425\text{dB.}$$

- To find the frequency point where the magnitude is -4.425dB,

sub w= jv in G(w) and find the magnitude of G(jv):

$$|G(jv)| = \frac{\sqrt{1+\left(\frac{v}{300}\right)^2}\ {}^{2}\!\sqrt{1+\left(\frac{v}{300}\right)^2}}{v\sqrt{1+v^2}}$$

- By trial and error, we find that v = 1.7 the magnitude is -4.4dB. We select this frequency to be the new gain crossover frequency v_c.

$$v_c = \frac{1}{\tau\sqrt{\alpha}} = 1.7 \text{ and we obtain } \tau = 0.9790$$

➢Determine lead compensator

$$G_D(w) = \frac{1+Tw}{1+\alpha Tw} = \frac{1+0.9790w}{1+0.3534w}$$

- Magnitude and phase angle curves for $G_D(jv)$ and magnitude and phase angles of the compensated open-loop transfer function $G_D(jv)$ $G(jv)$ are shown by solid curves.

From the bode diagram PM = 50° & GM = 14dB.

- Now the obtained controller transfer function will now transformed back to the z-plane by bilinear transformation.

$$w = \frac{2}{T}\frac{z-1}{z+1} = \frac{2}{0.2}\frac{z-1}{z+1} = 10\,\frac{z-1}{z+1}$$

$$G_D(z) = \frac{1 + 0.9790\left(10\dfrac{z-1}{z+1}\right)}{1 + 0.3534\left(10\dfrac{z-1}{z+1}\right)}$$

$$= \frac{2.3798z - 1.9387}{z - 0\ 5589}$$

The open-loop pulse transfer function of the compensated system is

$$G_D(z)G(z) = \frac{2.3798z - 1.9387}{z - 0\ 5589}\,\frac{0.03746(z + 0.9356)}{(z - 1)(z - 0\ 8187)}$$

$$= \frac{0.0891z^2 + 0.0108z - 0.0679}{z^3 - 2\ 3776z^2 + 1\ 8352z - 0\ 4576}$$

The closed-loop pulse transfer function of the designed system is

$$\frac{C(z)}{R(z)} = \frac{0.0891z^2 + 0.0108z - 0.0679}{z^3 - 2\ 2885z^2 + 1.8460z - 0.5255}$$

$$= \frac{0.0891(z + 0.9357)(z - 0.8145)}{(z - 0.8126)(z - 0.7379 - j0.3196)(z - 0.7379 + j0\ 3196)}$$

- From the closed-loop transfer function involves two zeros located at z= -0.9357 and z= 0.8145. the zero at z= 0.8145 almost cancels with the closed-loop pole at z= 0.8126.
- The effect of another zero at z= -0.9357 on transient and frequency response is very small. Since it located on the negative real axis of the z-plane between 0 and -1 and is close to point z=-1.
- The pair if complex conjugate poles acts as dominant closed-loop poles.