STABILITY ANALYSIS

DEFINITIONS ON STABILITY

a) **Zero state response:** The output response of a discrete data system, that is due to the input only is called the zero state response; all the initial conditions of the system are set to zero.

b) **Zero input response:** The output response of a discrete date system is due to the initial conditions only is called the zero input response; all the inputs of the system are set to zero.

From the principle of superposition, when a system is subjected to both inputs and initial conditions, the total output response is given by

Total response=Zero state response +zero input response

c) **Bounded input-bounded state stability:** Consider a linear time invariant discrete data system that is described by the following dynamic equations

$$\left.\begin{aligned} x(k+1) &= Gx(k) + Hu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}\right\}$$

The system is said to be bounded input bounded state (BIBS) stable if for any bounded input u(k), the state x(k) is also bounded.

d) **Bounded input- bounded output stability:** the system described by set of equations (6.1) is bounded input bounded output (BIBO) stable if for any bounded input, the output c(k) is also bounded. Since the output of a system is a linear combination of the state variables, a system that is BIBS stable must also be BIBO stable. However if the system is BIBO stable, it may or may not be BIBS stable.

e) **Zero input stability:** The system described by the dynamic equations is said to be zero input stable or simply stable if the zero input response c(k), subject to the finite initial conditions, reaches zero as k approaches infinity, otherwise , the system is unstable. Mathematically, zero input stability requires that

$$\left.\begin{aligned} &\|C(k)\| \le M < \infty \\ &\underset{k\to\infty}{Lt}\left|C_i(k)\right| = 0 \end{aligned}\right\}$$

Where $\|C(k)\|$=norm of a vector $x(k)$

M = a finite number

f) **Asymptotic stability:** The conditions given in equation are also the requirements

for asymptotic stability. Thus, zero input stability implies asymptotic stability.

g) **Zero input stability, asymptotic stability and characteristic roots:** For the linear discrete data system described by set of equations above, BIBO, Zero input and asymptotic stability all requires that the roots of the characteristic equation be inside the unit circle $|Z| = 1$ in the Z-plane.

MAPPING BETWEEN THE S-PLANE AND THE Z-PLANE

In the design of a continuous time control system, the location of the poles and zeros in the s-plane are very important in predicting the dynamic behavior of the system. Similarly, in designing discrete time control systems, the location of poles and zero in the z-plane are very important. Since the complex variables Z and S are related by $Z = e^{TS}$, the pole and zero locations in the z-plane are related to the pole and zero locations in the s-plane. Therefore, the stability of the linear time invariant discrete time closed loop system can be determined in terms of the locations of poles of the closed loop pulse transfer function.

When impulse sampling is incorporated into the process, the complex variables Z and S are related by the equation

$$Z = e^{TS}$$

With reference to location of the roots of the C.E in the S-domain, the imaginary axis i.e., $j\omega$ axis in the S-plane divides stable and unstable regions and the corresponding regions in the z-domain can be obtained by putting $S = \pm j\omega$ in equation and plotting the values of 'Z' thus obtained in another complex plane called z-plane.

$$\therefore Z = e^{\pm j\omega T} = \cos\omega T \pm j\sin\omega T$$

$$|Z| = 1, \angle Z = \pm\omega T$$

The above equation represents a circle of unit radius in the Z-plane as shown in figure

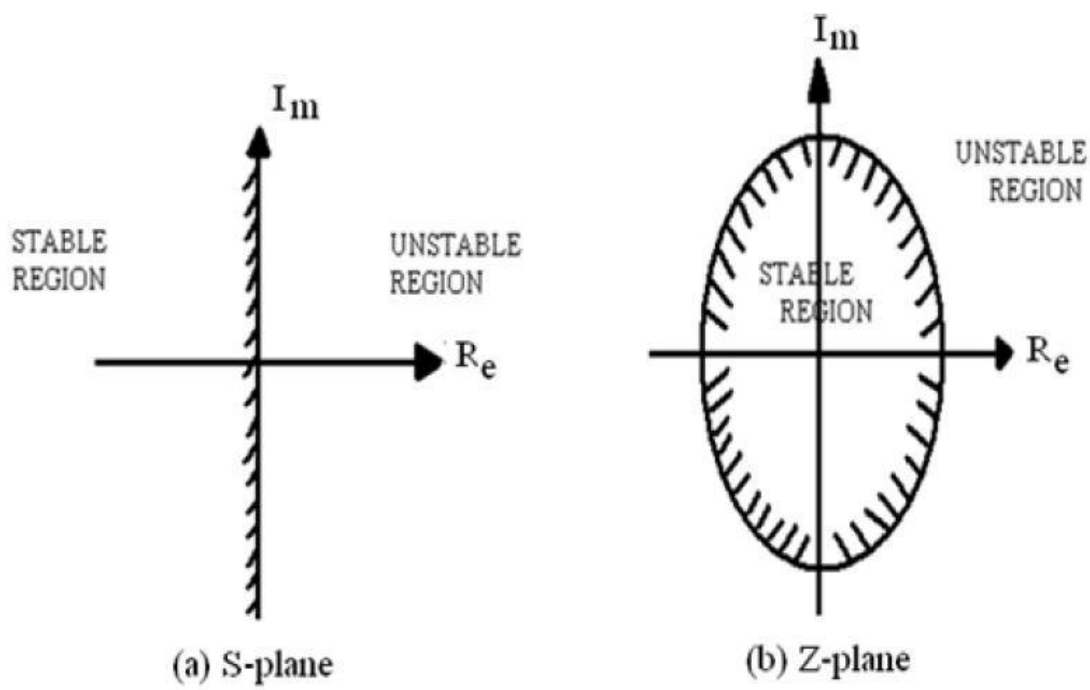

Fig : Mapping between s-plane and z-plane

The L.H.S of s-plane is mapped as the inside and R.H.S as outside of the unit circle in z-plane, which is verified below.

(a) Let $S = -\alpha \pm j\omega$, a point in the L.H.S of the s-plane. The corresponding point in the z-plane is given b

$$Z = e^{(-\alpha \pm j\omega)T} = e^{-\alpha T}\left(\cos\omega T \pm j\sin\omega T\right)$$

$$\therefore \left|Z\right| = e^{-\alpha T}, \angle Z = \pm\omega T$$

As α is the real part of the point under consideration lies in the L.H.S of the s-plane and T being positive $\left|Z\right| < 1$. Hence, the point $\left(-\alpha \pm j\omega\right)$ with negative real part located in s-plane lies insides the unit circle when mapped into z-plane.

(b) Let $S = \alpha \pm j\omega$, a point in the R.H.S of the s-plane. The corresponding point in the z-plane is given by

$$Z = e^{(\alpha \pm j\omega)T} = e^{\alpha T}\left(\cos\omega T \pm j\sin\omega T\right)$$

$$\therefore \left|Z\right| = e^{\alpha T}, \angle Z = \pm\omega T$$

As α , the real part of the point under consideration lies in the R.H.S of s-plane and T being positive $\left|Z\right| > 1$.Hence the point $\left(\alpha \pm j\omega\right)$ with positive real part located in s-plane lies outside the unit circle, when mapped into z-plane.

In view of the above analysis it is concluded that for the sampled data system to be stable all the roots of the Z-transformed characteristic equation, $1 + GH\left(Z\right) = 0$, should be located inside the unit circle centered at the origin in the z-plane and in case any root is located outside the unit circle centered at the origin in z-plane makes the system unstable.

PRIMARY STRIP AND COMPLEMENTARY STRIPS

We know that $\angle Z = \omega T$ the angle of Z varies from $-\infty$ to $+\infty$ as ω varies from $-\infty$ to ∞. Consider a representative point on the $j\omega$ axis in the s-plane. As this point moves from $-j\frac{1}{2}\omega_s$ to $j\frac{1}{2}\omega_s$ on the $j\omega$axis, where $\omega_s = \left(\frac{2\pi}{T}\right)$ is the sampling frequency, we have $|Z| = 1$, and $\angle Z$ varies from $-\pi$ to π in the counter clockwise direction in the z-plane. As the representative point moves from $j\frac{1}{2}\omega_s$ to $j\frac{3}{2}\omega_s$ on the $j\omega$ axis, the corresponding point in the z-plane traces out the unit circle once in the counter clockwise direction. Thus, as the point in the s-plane moves from $-\infty$ to ∞ on the $j\omega$ axis, we trace the unit circle in the z-plane an infinite number of times. From this analysis, it is clear that each strip of width ω_s in the left half of the s-plane maps into the inside of the unit circle in the z-plane. This implies that that the left half of the s-plane may be divided into an infinite number of periodic strips as shown in figure (6.3). The primary strip extends from $j\omega = -j\frac{1}{2}\omega_s$ to $j\frac{1}{2}\omega_s$. The complementary strips extend from

$j\frac{1}{2}\omega_s$ to $j\frac{3}{2}\omega_s$, $j\frac{3}{2}\omega_s$ to $j\frac{5}{2}\omega_s$,--------- and from $-j\frac{1}{2}\omega_s$ to $-j\frac{3}{2}\omega_s$, $-j\frac{3}{2}\omega_s$ to $-j\frac{5}{2}\omega_s$, ----

In the primary strip, if we trace the sequence of points 1-2-3-4-5-1 in the s-plane as shown by the numbers in figure above, then this path is mapped into the unit circle centered at the origin of the z-plane, as shown in figure (b). The corresponding points 1,2,3,4 and 5 in the z- plane are shown by the numbers in figure (b).

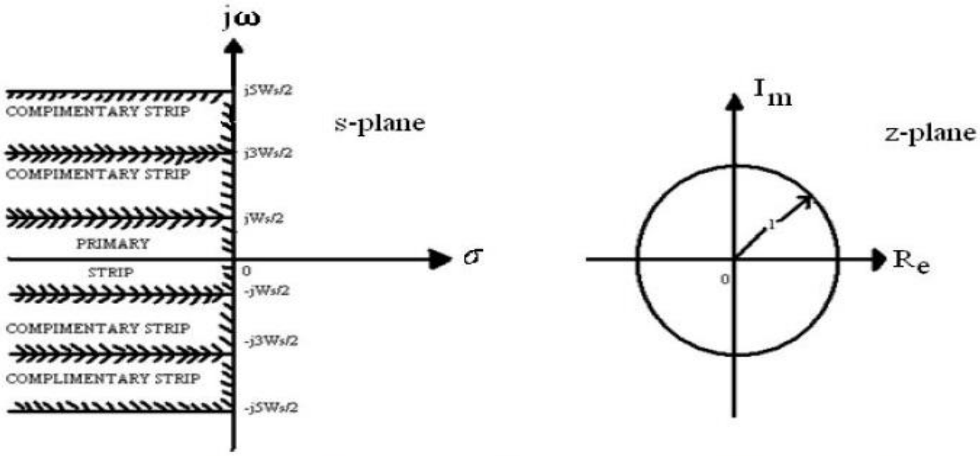

Fig : Periodic strips in the s-plane and the corresponding region(unit circle centered at the origin) in the s-plane

The area enclosed by any of the complimentary strips is mapped into the same unit circle in the z-plane. This means that the correspondence between the z-plane and s-plane is not unique. A point in the z-plane corresponds to as infinite number of points in the s-plane, although a point in the s-plane corresponds to a single point in the z-plane. Since the entire left half of the s-plane is mapped into the interior of the unit circle in the z-plane, the entire right half of the s-plane is mapped into the exterior of the unit circle in the z-plane. As mentioned earlier, the j**ω** axis in the s-plane maps into the unit circle in the z-plane.

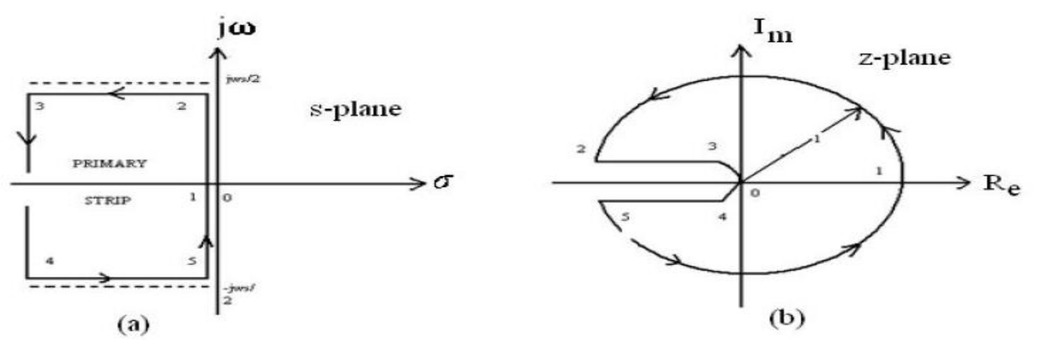

Fig: Diagrams showing the correspondence between the primary strip in the s-plane and the unit circle in the z-plane (a) A Path in s-plane (b) The corresponding path in the z plane

METHODS FOR TESTING STABILITY

The following three tests are used for analyzing the stability of a discrete time control system.

- Bilinear transformation or extended RH criterion.
- Jury's stability test.
- Schur-chon criterion.

Bilinear Transformation

By the bilinear transformation, it was found that the stability of a discrete time systems can be determined without finding the actual numerical values of the roots of the characteristic equation.

One such transformation defined by

$$Z = \frac{w+1}{w-1}$$

Which, when solved for w, gives

$$w = \frac{Z+1}{Z-1}$$

maps the inside of the unit circle in the z-plane into the left half of the ω-plane. This can be seen as follows. Let $w = \alpha + j\beta$

Since the inside of the unit circle in the z-plane is

$$|Z| = \left|\frac{w+1}{w-1}\right| = \left|\frac{\alpha + j\beta + 1}{\alpha + j\beta - 1}\right| < 1$$

$$\Rightarrow \frac{(\alpha+1)^2 + \beta^2}{(\alpha-1)^2 + \beta^2} < 1$$

$$\Rightarrow (\alpha+1)^2 + \beta^2 < (\alpha-1)^2 + \beta^2$$

$$\Rightarrow \alpha < 0$$

Thus, the inside of the unit circle in the z-Plane ($|Z|<1$) corresponds to the left half of the w-plane. The unit circle in the z-plane is mapped into the imaginary axis in the w-plane, and the outside of the unit circle in the z-plane is mapped into right half of the w-plane.

In the stability analysis using the bilinear transformation coupled with the Routh Stability criterion, we first substitute $\frac{w+1}{w-1}$ for Z in the characteristic equation

$$P(z) = a_0 Z^n + a_1 Z^{n-1} + - - - - - - - - + a_{n-1} Z + a_n = 0$$

as follows

$$a_0\left(\frac{w+1}{w-1}\right)^n + a_1\left(\frac{w+1}{w-1}\right)^{n-1} + - - - - - - - - - - - - + a_{n-1}\left(\frac{w+1}{w-1}\right) + a_n = 0$$

$$\Rightarrow Q(\omega) = b_0 w^n + b_1 w^{n-1} + - - - - - - - - - - - - - - + b_{n-1} w + b_n = 0$$

Once we transform P(z)=0 into Q(w)=0, it is possible to apply the Routh's stability criterion in the same manner as in continuous time systems.

It is noted that the bilinear transformation coupled with the Routh's stability criterion will indicate exactly how many roots of characteristic equation lie in the right half of the w-plane and how many lie on the imaginary axis.

Problem-1: Analyze the stability of the following systems by using bilinear transformation.

(a) $P(z) = Z^3 + 3.3Z^2 + 3Z + 0.8 = 0$

Put $Z = \frac{w+1}{w-1}$

$$\left(\frac{w+1}{w-1}\right)^3 + 3.3\left(\frac{w+1}{w-1}\right)^2 + 3\left(\frac{w+1}{w-1}\right) + 0.8 = 0$$

$$(w+1)^3 + 3.3(w+1)^2(w-1) + 3\,(w+1)(w-1)^2 + 0.8(w-1)^3 = 0$$

$$w^3 + 1 + 3w^2 + 3w + 3.3(w^2-1)(w+1) + 3(w^2-1)(w-1) + 0.8(w^3 - 1.3w^2 + 3w) = 0$$

$$Q(w) = 8.1w^3 + 0.9w^2 - 0.93w - 0.1 = 0$$

$$
\begin{array}{c|cc}
w^3 & 8.1 & -0.9 \\
w^2 & 0.9 & -0.1 \\
w^1 & 0 & 0 \\
w^0 & &
\end{array}
\leftarrow \text{Routh's test breaks down}
$$

$$A(w) = 0.9w^2 - 0.1$$

$$\frac{dA}{dw} = 1.8w$$

$$
\begin{array}{c|cc}
w^3 & 8.1 & -0.9 \\
w^2 & 0.9 & -0.1 \\
w^1 & 1.8 & 0 \\
w^0 & -0.1 & 0
\end{array}
$$

Since one sign change occurs in the first column of Routh's tabulation, the characteristic equation has one root in the right half of ω-plane (or) $\mathrm{P}(z)$ has one root outside the unit circle in the z-plane.

(b) $P(z) = Z^3 + Z^2 + Z + 1 = 0$

The roots of P(z) are at Z=1,Z=j and Z=-j, which are all on the unit circle, thus the system is unstable.

The Jury Stability Test

In applying the Jury stability test to a given characteristic equation P(z)=0, we construct a stable whose elements are based on the coefficients of P(z). Assume that the characteristic equation P(z) is a polynomial in Z as follows:

$P(z) = a_0 Z^n + a_1 Z^{n-1} + - - - - - - - - + a_{n-1} Z + a_n$

Where $a_0 > 0$. Then the Jury table becomes as given below

Row	Z^0	Z^1	Z^2	Z^3	....	Z^{n-2}	Z^{n-1}	Z^n
1	a_n	a_{n-1}	a_{n-2}	a_{n-3}	....	a_2	a_1	a_0
2	a_0	a_1	a_2	a_3	....	a_{n-2}	a_{n-1}	a_n
3	b_{n-1}	b_{n-2}	b_{n-3}	....	....	b_1	b_0	
4	b_0	b_1	b_2	....	....	b_{n-2}	b_{n-1}	
5	c_{n-2}	c_{n-3}	c_{n-4}	....	....	c_0		
6	c_0	c_1	c_2	....	....	c_{n-2}		
.	.	.	.			.		
.	.	.	.			.		
.	.	.	.			.		
.	.	.	.			.		
.	.	.	.			.		
$2n-5$	p_3	p_2	p_1	p_0				
$2n-4$	p_1	p_1	p_2	p_3				
$2n-3$	q_2	q_1	q_0					

Where

- The first two rows consists of the coefficients of P (z) arranged in ascending order of powers of Z in first row and in reverse order in next row.
- All even numbered rows are the reverse of the preceding row.
- Elements of row three through (2n-3) are calculated as

$$b_k = \begin{vmatrix} a_n & a_{n-1-k} \\ a_0 & a_{k+1} \end{vmatrix} \text{, k= 0, 1, 2………, n-1}$$

$$c_k = \begin{vmatrix} b_{n-1} & b_{n-2-k} \\ b_0 & b_{k+1} \end{vmatrix} \text{, k= 0, 1, 2………, n-2}$$

. .

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. .

. .

.   .

.   .

$$q_k = \begin{vmatrix} p_3 & p_{k-2} \\ p_0 & p_{k+1} \end{vmatrix} \text{ , k= 0, 1, 2}$$

Procedure is continued until (2n-3)rd row is reached which contains exactly three elements.

The system is said to be stable if it satisfies the following conditions.

i. $|a_n| < |a_0|$

ii. $P(z)\big|_{z=1} > 0$

iii. $P(z)\big|_{z=-1} \begin{cases} > 0 \ for\ n\ even \\ < 0 \ for\ n\ odd \end{cases}$

iv. $|b_{n-1}| > |b_0|$

$|c_{n-2}| > |c_0|$

.   .

.   .

.   .

$|q_2| > |q_0|$

Problem-2: Analyze the stability of the following systems by using Jury's stability test

$(a)\quad P(z) = Z^3 - 1.25Z^2 - 1.375Z - 0.25 = 0$

Solution: $P(z) = Z^3 - 1.25Z^2 - 1.375Z - 0.25 = 0$ --- (1)

From the given characteristic equation

$a_0 = 1, a_1 = -1.25, a_2 = -1.375, a_3 = -0.25$

i). $|a_3| < |a_0| i.e |-0.25| < |1|$

ii) $P(1)$=1-1.25-1.375-0.25=-1.875<0

iii). P (-1) = -1-1.25+1.375-0.25<0 for n-odd

Since P(1) is negative, not all the roots of equation(1) are inside the unit circle, and the system is unstable. So there is no need to carry out the Jury's tabulation.

(b) $P(z) = Z^4 - 1.2Z^3 + 0.07Z^2 + 0.3Z - 0.08 = 0$

Solution: $P(z) = Z^4 - 1.2Z^3 + 0.07Z^2 + 0.3Z - 0.08 = 0$ --- (2)

From the given characteristics equation

$a_0 = 1, a_1 = -1.2, a_2 = 0.075, a_3 = 0.3, a_4 = -0.08$

i) $|a_4| < |a_0|$ $i.e$ $|-0.08| < |1|$

ii) P (1) = 1-1.2+0.07+0.3-0.08= 0.09>0

iii) P (-1) = 1+1.2+0.07-0.3-0.08= 1.89>0, n=4=even

The first three conditions for stability are satisfied, we have to check for the fourth condition.

Row	Z^0	Z^1	Z^2	Z^3	Z^4
1	-0.08	0.3	0.07	-1.2	1
2	1	-1.2	0.07	0.3	-0.08
3	b_3 -0.994	b_2 1.176	b_1 -0.0756	b_0 -0.204	
4	-0.204	-0.0756	1.176	-0.994	
5	0.946	-1.184	0.315		

$b_0 = 1.2 \times 0.08 - 0.3 \times 1 = -0.204$

$b_1 = -0.07 \times 0.08 - 0.07 = -0.0756$

$b_2 = -0.08 \times 0.3 + 1.2 \times 1 = 1.176$

$b_3 = 0.08^2 - 1^2 = 0.994$

iv) $|b_3| > |b_0|$ *i.e.* $|0.994| > |0.204|$

$|c_2| > |c_0|$ *i.e.* $|0.946| > |0.315|$

Thus fourth condition is satisfied. Since all conditions for stability are satisfied, the given system is stable, or all roots lie inside the unit circle in the z-plane.

$(c)\quad P(z) = Z^3 + 3.3Z^2 + 4Z + 0.8 = 0$

Solution: $P(z) = Z^3 + 3.3Z^2 + 4Z + 0.8 = 0$ --- (3)

From the given characteristics equation

$a_0 = 1, a_1 = 3.3, a_2 = 4, a_3 = 0.8$

i) $|a_3| < |a_0|$ *i.e* $|0.8| < |1|$

ii) P (1) = 1+3.3+4+0.8= 9.1>0

iii) P (-1) = -1+3.32-4+8= -0.9<0, n=3=odd

The first three conditions for stability are satisfied, we have to check for the fourth condition.

Row	Z^0	Z^1	Z^2	Z^3
1	0.8	4	3.3	1
2	1	3.3	4	0.8
3	b_2 -0.36	b_1 -0.1	b_{01} -1.36	
4	-1.36	-0.1	-0.36	

$$b_0 = 3.3 \times 0.8 - 4 \times 1 = -1.36$$

$$b_1 = 4 \times 0.8 - 3.3 \times 1 = -0.1$$

$$b_2 = 0.8 \times 0.8 - 1 \times 1 = -0.36$$

iv) $|b_2| > |b_0|$ *but* $|0.36| < |1.36|$. Thus fourth condition is not satisfied. So equation.3 has at least one root outside the unit circle.

Problem-3: Consider the discrete time unity-feedback control system, whose open loop pulse transfer function is given by $G(z) = \dfrac{K(0.3679Z + 0.2642)}{(Z - 0.3679)(Z - 1)}$

Determine the range of gain K for stability by use of the Jury's stability test.

Solution: The characteristic equation of the system is

$$P(z) = 1 + G(z)H(z) = 0$$

$$= 1 + \frac{K(0.3679Z + 0.2642}{(Z - 0.3679Z)(Z - 1)} = 0$$

$$= Z^2 - 1.3679Z + 0.3679 + K(0.3679Z + 0.2642) = 0$$

$$\Rightarrow P(z) = Z^2 + 0.3679K - 1.3679)Z + (0.3679 + 0.2642K) = 0$$

Since this is a second order system, the Jury's stability conditions may be written as follows:

i) $|a_2| < |a_0|$

ii) P (1)>0

iii) P (-1)>0, n=2=even

We shall now apply the first condition for stability. Since $a_2 = 0.3679 + 0.2642K$ and $a_0 = 1$, the first condition for stability becomes

$$|0.3769 + 0.2642K| < 1$$

i .e. 0.3769+0.2642K< ± 1

$$\text{i.e. } K = \frac{1-0.3649}{0.2642} \text{ (or) } \frac{-1-0.3649}{0.2642}$$

=2.3925 (or) -5.775

The second condition for stability becomes

P (1) = 1+ (0.3679K-1.3679) +0.3679+0.2642K>0

$$\Rightarrow 0.6321K > 0$$

$$\Rightarrow K > 0$$

The third condition for stability becomes

P (-1) = 1-(0.3679K-1.3679) + 0.3679+0.2642K>0

$$\Rightarrow 2.7358 - 0.1037K > 0$$

$$\Rightarrow K < \frac{2.735}{0.1037} < 26.374$$

$\therefore$ For stability, the constant K is 0 < K < 2.3925

Problem-4: Consider the system described by

y (k) - 0.6y (k-1) – 0.81y (k-2) +0.67y (k-3) – 0.12y (k-4) = x (k)

where x (k) is the input and y (k) is the output of the system. Determine the stability of the system.

Solution:

y (k) - 0.6y (k-1) – 0.81y (k-2) +0.67y (k-3) – 0.12y (k-4) = x (k)

By taking Z-transforms on both sides

Y(z)-0.6Z^{-1} Y(z)-0.81 Z^{-2} Y(z)+0.67 Z^{-3} Y(z) -0.12 Z^{-4} Y(z) =X(z)

$$\frac{Y(z)}{X(z)} = \frac{1}{1-0.6Z^{-1}-0.81Z^{-2}+0.67Z^{-3}-0.12Z^{-4}}$$

$$= \frac{Z^4}{Z^4-0.6Z^3-0.81Z^2+0.67Z-0.12}$$

The characteristic equation of the system is

$$P(z) = Z^4-0.6Z^3-0.81Z^2+0.67Z-0.12 = 0$$

From the above equation

$$a_0 = 1, a_1 = -0.6, a_2 = -0.81, a_3 = 0.67, a_4 = -0.12$$

i) $|a_4| < |a_0|\ i.e\ |0.12| < |1|$

ii) P (1) = 1-0.6-0.81+0.67-0.12 = 0.14>0

iii) P (-1) = 1+0.6-0.81-0.67-0.12 = 0

The condition is not satisfied. P (-1) = 0 implies that there is one root at Z = -1

The first three conditions for stability are checked. We have to test for the fourth condition.

Row	Z^0	Z^1	Z^2	Z^3	Z^4
1	-0.12	0.67	-0.81	-0.6	1
2	1	-0.6	-0.81	0.67	-0.12
3	-0.9856	0.52	0.907	-0.598	
4	-0.598	0.907	0.52	-0.9856	
5	0.614	0.03	-0.583		
6	-0.583	0.03	0.614		

iv) $|b_3| > |b_0|\ i.e. |-0.9856| > |-0.598|$

$|c_2| > |c_0|\ i.e. |0.614| > |-0.5834|$

Thus the fourth condition is satisfied.

From the preceding analysis, we can conclude that the characteristic equation P(z) = 0 involves a root at Z = -1 and the other three roots are in the unit circle centered at the origin of the z-plane. So the system is critically stable.

Problem-5: Consider the following characteristic equation

$$P(z) = Z^3 - 1.3Z^2 - 0.08Z - 0.24 = 0$$

Determine whether or not any of the roots of the characteristic equation lie outside the unit circle in the z-plane. Use the bilinear transformation and the Routh stability criterion.

Solution: Let $Z = \frac{w+1}{w-1}$

The given characteristic equation becomes

$$\left(\frac{w+1}{w-1}\right)^3 - 1.3\left(\frac{w+1}{w-1}\right)^2 - 0.08\left(\frac{w+1}{w-1}\right) + 0.24 = 0$$

$$(w+1)^3 - 1.3(w+1)^2(w-1) - 0.08(w+1)(w-1)^2 + 0.24(w-1)^3 = 0$$

$$w^3 + 1 + 3w^2 + 3w - 1.3(w^2-1)(w+1) - 0.08(w^2-1)(w-1) + 0.24(w^3 - 1 - 3w^2 + 3w) = 0$$

$$-0.14w^3 + 1.06w^2 + 5.1w + 1.98 = 0$$

$$w^3 - 7.57w^2 - 36.43w - 14.14 = 0$$

The Routh array for the above equation is

$$\begin{array}{c|cc} w^3 & 1 & -36.43 \\ w^2 & -7.57 & -14.14 \\ w^1 & -38.30 & 0 \\ w^0 & -14.14 & \end{array} \leftarrow signchange$$

There is one sign change in the first column of Routh's array and hence there is one root in the right half of the w-plane. This means that the original characteristic equation has one root outside the unit circle in the z-plane. So the system is unstable.

Problem-6: A discrete time system

x(k+1)=Ax(k) +Bu(k) has the system matrix $A=\begin{bmatrix}1 & a\\2 & 1/2\end{bmatrix}$. For what value 'a' is the system stable ?

Solution: The characteristic equation of the system is

$$|ZI-G|=0$$

$$\left|\begin{bmatrix}Z & 0\\0 & Z\end{bmatrix}-\begin{bmatrix}1 & a\\2 & 0.5\end{bmatrix}\right|=0$$

$$\begin{vmatrix}Z-1 & -a\\-2 & Z-0.5\end{vmatrix}=0$$

(Z-1)(Z-0.5)-2a = 0

$$\therefore P(z)=Z^2-1.5Z+0.5-2a=0$$

$$\Rightarrow a_0=1, a_1=-1.5, a_2=0.5-2a$$

Since this is a second order system, the Jury's stability conditions may be written as follows.

i) $|a_2|<|a_0|$

ii) P (1)>0

iii) P (-1)>0, n=2=even

The first condition for stability becomes

$|0.5-2a|<1$

0.5-2a<± 1

0.5-2a<-1 (or) 0.5-2a<1

-2a<-1.5 (or) -2a<0.5

A< 0.75 (or) a>-0.25

The second condition for stability becomes

$P(1) = 1-1.5+0.5-2a>0$

$\Rightarrow -2a>0 \;\; \Rightarrow a<0$

The third condition for stability becomes

$P(-1) = 1+1.5+0.5-2A>0$

$\Rightarrow 3-2a>0$

$\Rightarrow -2a>-3 \Rightarrow a<1.5$

$\therefore$ The condition for stability is $\quad -0.25 < a < 0$